MOREY'S ARITHMETICS

INTERMEDIATE ARITHMETIC

CHARLES SCRIBNER'S SONS

INTERMEDIATE ARITHMETIC

MOREY'S ARITHMETICS

INTERMEDIATE ARITHMETIC

BY

CHARLES W. MOREY, M.A.

MASTER OF HIGHLAND SCHOOL
LOWELL, MASSACHUSETTS

NEW YORK
CHARLES SCRIBNER'S SONS
1911

PREFACE

THE object of the series of text-books of which Intermediate Arithmetic is a part is threefold:—

I. To secure accuracy and facility in the mechanics of number. To accomplish this result there is a continuous presentation of the fundamental processes with integers and with fractions, both common and decimal.

II. To relate the use of number to the affairs of everyday life. This requires the solution of problems in household and school economics, in the affairs of the playground, the workshop, and the farm, and a study of conditions existing in the business world.

III. To develop a power of insight and analysis which shall render pupils resourceful and self-reliant. This involves the introduction of many problems requiring the use of more than a single process in their solution, and drawn from a great variety of sources, both within and without the experience of the pupil. A foundation is thus laid for work in mathematics beyond grammar school grades.

Some subjects ordinarily included in grammar school courses are purposely omitted because it is believed that the mastery of a few subjects is the sane and wise method of procedure in the development of mathematical power. Quality, not quantity, of work is of prime importance in training young pupils.

The subject matter of Intermediate Arithmetic is so arranged that pupils leaving school before the completion of the regular course are given opportunity to acquire some knowledge of practical business methods.

The author wishes to acknowledge his indebtedness to all who have assisted in the preparation of the manuscript, and especially to Mr. Myron T. Pritchard, Master of the Everett School, Boston, Massachusetts, for wise counsel and criticism.

C. W. M.

CONTENTS

INTERMEDIATE ARITHMETIC

PART I

NOTATION AND NUMERATION

1. How many units make 1 ten? How many tens make 1 hundred? How many hundreds make 1 thousand?

2. The middle 3 in the number 333 represents how many times as many units as the right-hand 3?

3. The left-hand 3 represents how many times as many units as the right-hand 3.

Each figure in a number has a value determined by its place in the number.

4. Compare the value of the 2's in 22; 202; 220; 2200; 2020; 2002.

5. Using 4's and 0's write a number in which one 4 represents one hundred times as many as the other 4.

Separate into groups, and read:

6. 8067	**11.** 20387	**16.** 480465	**21.** 1378543
7. 9350	**12.** 58706	**17.** 896302	**22.** 5490876
8. 7006	**13.** 93042	**18.** 107069	**23.** 9040732
9. 8360	**14.** 10087	**19.** 316400	**24.** 27438564
10. 6040	**15.** 80649	**20.** 300602	**25.** 764312857

26. When we separate numbers into groups of three figures each, what is the right-hand group called? The next group to the left? The next group?

Write in figures:

1. Three thousand forty.
2. Seventeen thousand nine hundred twenty-six.
3. Sixty thousand six hundred six.
4. One hundred thirty-nine thousand.
5. One hundred thousand thirty-nine.
6. Three hundred four thousand one hundred ten.
7. Eight hundred twenty thousand twenty-four.
8. One million two hundred twelve thousand.
9. Three million forty-six thousand seventeen.
10. Two hundred sixty-seven million eight hundred four thousand seventy-six.

ROMAN NOTATION AND NUMERATION

Letters used	I	V	X	L	C	D	M
Values	1	5	10	50	100	500	1000

By combining these letters we can express any number by following these rules:

I. When a letter is followed by the same letter or by one of less value, add the values of the letters. Thus, XX = 20; XIII = 13.

II. When a letter is followed by one of greater value, subtract the letter of less value from the letter of greater value. Thus, IX = 9; XL = 40.

Read:

1. XIX XXXVII LXV CIV DC

Write in Roman notation:

2. 8 14 25 43 52 66 78 81 99

DRILL IN FUNDAMENTAL PROCESSES

NOTE. Each exercise should begin with a short, rapid oral drill in the fundamental processes. This daily drill should be continued until accuracy and facility render such work unnecessary.

ADDITION *Oral*

Add 2 to each number:

3	1	5	2	7	0	4	8	6	9

Add 4; 6; 8; 1; 3; 9; 5; 7.

Addition is the process of uniting two or more numbers into one number.

The *sum* or *amount* is the result of addition.

DRILL TABLE

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
1.	35	28	36	52	61	44	70	86	91	60
2.	20	12	78	37	53	62	45	71	87	92
3.	93	21	13	79	38	54	63	46	72	88
4.	10	30	22	14	80	39	55	64	47	73
5.	74	94	31	23	15	81	48	56	65	29
6.	98	59	40	32	24	16	82	95	57	66
7.	89	75	67	41	33	25	17	83	49	58
8.	99	68	76	50	42	34	26	18	84	97
9.	69	96	90	77	51	43	11	27	19	85

Add 2 to each number; add 3; 4; 5; 6; 7; 8; 9.

Add 20 to each number; add 30; 40; 50; 60; 70; 80; 90.

Give the sum of each number and any number of two figures. Thus, 35 + 78. This means 35 + 70 + 8. Think 35, 105, 113. Say 113.

Find the sum of each column. Of each row.

ORAL PROBLEMS

1. Miriam used her weekly allowance as follows: 7 cents for candy, 2 cents for a pencil, 6 cents for flower seeds, 5 cents for a soda, and 5 cents for the school savings bank. How much was her weekly allowance?

2. At the playground 15 boys enter the potato race, 12 the three-legged race, and 9 the running race. How many boys in the three races?

3. How much did it cost Sarah to go to the picnic, if she spent 20 cents for car fares, 5 cents for lemonade, 15 cents for a steamer ride, and 10 cents on the merry-go-round?

4. Mr. Kennedy buys Harold a knife for 25 cents, Frank a box of crayons for 20 cents, and Alice a doll for 50 cents. How much does he pay for all?

5. Mrs. Hovey canned 16 jars of blueberries, 9 jars of raspberries, 11 jars of strawberries, and 8 jars of cherries. How many jars in all?

6. We sold from our garden 6 bushels of pears, 2 bushels of plums, 13 bushels of apples, and 3 bushels of grapes. How many bushels of fruit did we sell?

7. John bought a hat for 3 dollars, a coat for 12 dollars, a pair of shoes for 4 dollars, and collars and cuffs for 1 dollar. How much did he pay for all?

8. A farmer brings us a dozen ears of corn for 12 cents, two boxes of blueberries for 25 cents, and a dozen eggs for 40 cents. How much do all cost?

9. Fred entered the primary school when he was 6 years old. He spent 3 years in the primary school, 5

years in the grammar school, 4 years in the high school. How old was he when he graduated from the high school?

10. At the settlement house there are 13 girls in the dressmaking class, 17 in the millinery class, and as many in the cooking class as in both the other classes. How many in the cooking class? How many in the three classes?

SUBTRACTION *Oral*

Take 4 from:

10	13	16	19	11	14	17	12	15	18

Take 3; 6; 9; 1; 5; 8; 2; 7.

Subtraction is the process of taking one number from another, or of finding the difference between two numbers.

The *minuend* is the number from which something is taken.

The *subtrahend* is the number taken from the minuend.

The *remainder* or *difference* is the result of subtraction.

Take 2 from each number in the table on page 227. Take 3; 4; 5; 6; 7; 8; 9.

From 100 take each of the numbers in the table. Thus, $100 - 57 = 100 - 50 - 7$. Think 100, 50, 43. Say 43.

Give differences between any number of two figures and the numbers in the table.

ORAL PROBLEMS

1. Six pupils out of a class of 40 were not promoted. How many were promoted?

2. Frank earned 25 cents on Monday and 9 cents less on Tuesday. How much did he earn on Tuesday?

3. Out of a flock of 37 chickens, a hawk caught 3 and 8 died. How many were left?

4. Joe sells 43 papers and Sam 15 less. How many does Sam sell?

5. A party of 45 people started to climb Mt. Adams; 19 went half way up. How many reached the top?

6. In a box of 3 dozen eggs 9 were broken. How many were good?

7. There were 30 men and 50 women in a hospital. How many patients were there after 40 were discharged as cured?

8. What is the change from a 50-cent piece given in payment for oranges for 18 cents, tomatoes for 8 cents, and lettuce for 5 cents?

9. A party of 50 children went on a picnic down the river; 18 of them went on the boat, and the rest on the cars. How many went on the cars?

10. I gave a two-dollar bill to pay for a 75-cent cap. What was my change?

MULTIPLICATION *Oral*

Multiply by 2:

3	7	5	0	9	2	4	6	8

Multiply by 3; 4; 5; 6; 7; 8; 9; 10; 11; 12.

Multiplication is the process of combining several *equal* numbers into one number.

The *multiplicand* is one of the equal numbers. This is the number to be multiplied.

The *multiplier* is the number by which we multiply. It shows how many times the multiplicand is to be taken.

The *product* is the result of multiplication.

Multiply by 4 the numbers in the table on page 227. Thus, 68 multiplied by 4: $68 = 60 + 8$. $4 \times 60 = 240$; $4 \times 8 = 32$; $240 + 32 = 272$.

Multiply by 2; 3; 5; 6; 7; 8; 9.

ORAL PROBLEMS

1. If a steamer makes a 2-mile trip 6 times every day, how many miles does it run in a week?

2. If 2 pears are sold for 5 cents, what will 20 cost?

3. What will Ella's vacation of 3 weeks cost her, if she pays 8 dollars a week for board and 4 dollars each week for laundry and other expenses?

4. What will 24 oranges and 12 lemons cost at 25 cents a dozen?

5. How many children in the march if there are 4 lines and 15 children in each line?

6. What must I pay for 5 melons at 6 cents apiece and 2 boxes of berries at 12 cents a box?

7. Mr. Hubbard brings vegetables to the city twice a week. He lives 7 miles away. How many miles does he travel each week?

8. What will $\frac{1}{2}$ dozen bananas and 4 apples cost at 3 cents apiece?

9. How much will 5 packages of cereal cost at 15 cents a package?

10. Grace sends 8 Christmas cards. If she pays 5 cents for each card, 1 cent for each envelope, and puts a 2-cent stamp on each envelope, how much does she pay for all?

DIVISION *Oral*

Division is the process of finding how many times one number is contained in another number, or of finding one of the equal parts of a number.

The *dividend* is the number to be divided.

The *divisor* is the number by which we divide.

The *quotient* is the result of division.

Divide by 2 the numbers in the table on page 227; divide by 3; 4; 5; 6; 7; 8; 9; 10; 11; 12.

ORAL PROBLEMS

1. I have 84 pounds of salt. How many 7-pound packages can I make from it?

2. How many feet long is a steel rod that measures 108 inches?

3. How many berries at 12 cents a box must Ralph sell to earn a football worth $1.20?

4. Mrs. Miller sold the grocer 2 dozen eggs at 30 cents a dozen and took her pay in sugar at 6 cents a pound. How many pounds did she receive?

5. John had 50 cents. He lost 8 cents, and spent the rest for firecrackers at 6 cents a bunch. How many bunches did he buy?

6. Mr. Fisher earns 2 dollars a day. How long will it take him to earn 72 dollars?

7. How many calls does a district nurse average a week if she makes 160 in 4 weeks?

8. Carrie pledged $1 to the children's aid society. How long will it take her to pay it, if she earns 15 cents every week and her mother gives her 5 cents every week?

9. Lucy spends 10 days of her vacation at the seashore, 14 days in the country, and 4 days at home. How many weeks is her vacation?

10. Eight girls have a sale of fancy articles. They pay $2 for advertising and $3 for other expenses. They take in $61. What is each girl's share of the profits?

UNITED STATES MONEY *Oral and Written*

1. Read: $4.00; $6.00; $2.40; $1.08; $0.27; $0.20; $0.05.

2. How many cents make one dollar? How many cents in $2.00? $3.00? $2.50? $1.67? $1.07?

3. How many dollars in 500 cents? 600 cents? 800 ¢? 1000 ¢?

4. Write as dollars and cents: 125 cents; 260 cents; 308 ¢; 203 ¢.

5. Write with the dollar sign: 25 cents; 60 cents; 4 ¢; 1 ¢.

Remember in addition and subtraction to place the decimal points one under another. Why?

6. Add: $8.04, $3.17, $2.80, $7.05, $9.62.

7. Add: $0.08, $0.56, $0.47, $0.40, $0.83, $0.05.

8. Add: $3.00, $3.30, $3.03, $0.30, $0.33, $0.03.

9. Take \$5.19 from \$8.43.
10. From \$0.87 take \$0.60.
11. Take \$4.70 from \$6.75.
12. From \$0.50 take \$0.08.
13. Take \$3.08 from \$4.00.
14. From \$0.90 take \$0.84.
15. Take \$2.25 from \$5.00.
16. From \$2.00 take \$0.05.
17. Take \$3.80 from \$6.20.
18. From \$6.25 take \$3.80.

Multiply:

19. \$5.12	20. \$3.02	21. \$1.05	22. \$0.12	23. \$0.18
4	3	4	3	6
24. \$0.60	25. \$0.25	26. \$1.75	27. \$0.02	28. \$0.02
8	4	2	5	3

Divide:

29. 2)\$3.12	30. 3)\$1.56	31. 4)\$4.20	32. 3)\$0.75
33. 4)\$4.80	34. 9)\$1.08	35. 5)\$1.50	36. 3)\$0.09
37. 8)\$0.24	38. 5)\$2.00	39. 6)\$4.20	40. 7)\$0.84

DRILL IN ADDITION AND SUBTRACTION *Written*

Add, and test your work:

1.	2.	3.	4.	5.
48	184	1789	11,829	235,807
79	276	1642	63,476	176,943
96	381	8959	78,259	379,870
85	768	7749	51,987	135,560
58	295	9801	21,684	874,396
78	429	7395	30,906	792,381
99	973	4728	15,897	547,869

6.	7.	8.	9.	10.
$0.74	$678	$0.87	68	96
.08	7	37.56	706	8453
.76	8	.17	9083	473,584
.09	803	.08	67,384	6708
.58	49	9.04	307	403
.29	28	.28	26,308	27
.90	7	57.01	49	8

Find the difference, and test your work:

1. $34.65 – $6.80
2. 7623 – 930
3. $12,500 – $6700
4. $58.34 – $20.70
5. 8542 – 3719
6. 32,706 – 10,834
7. $43.52 – $17.56
8. 3627 – 2864
9. 17,280 – 12,780
10. $27.90 – $18.25
11. 5625 – 4096
12. 35,060 – 12,087
13. $34.20 – $15.05
14. 8070 – 4308
15. 67,324 – 34,827
16. From 8000 take 8; 80; 800; 88; 880; 808; 888.

	Minuend	Subtrahend	Remainder		Minuend	Subtrahend	Remainder
17.	?	$6.95	$1.38	18.	722	266	?
19.	$8.00	$3.69	?	20.	?	392	827
21.	$5.23	?	$3.65	22.	648	?	209
23.	?	$5.26	$0.79	24.	900	253	?
25.	$4.60	$1.87	?	26.	?	539	278
27.	$9.05	?	$3.88	28.	753	?	167

Note. There should be frequent dictation of numbers to be added and subtracted.

PROBLEMS *Written*

1. One lot of cloth contained 850 yards, another 1285 yards, and a third 1460 yards. How many yards in all?

2. An iceman cut 2250 tons of ice. How much had he left after selling 1780 tons?

3. A farmer raised 375 bushels of corn in one year, and in the next year 250 bushels more than in the first year. How many bushels did he raise in both years?

4. Mr. Morse bought a house for $2800, and another for $3650. He sold both for $7290. How much did he gain?

5. Mr. Cook paid $1096 for a house lot and on it built a house for $3265. He sold both at a gain of $475. How much did he receive?

6. Mr. Wright's bank account showed a deposit of $1296 on Monday morning. On Monday he deposited $582 and withdrew by check $653; on Tuesday he deposited $498 and withdrew $379; on Wednesday he deposited $889 and withdrew $1498; on Thursday he deposited $756. What were his total deposits? How much had he to his credit on Friday morning?

7. A butcher's charges against a family for one week were $1.37, $0.69, $2.08, $0.87, $1.75, and $0.98. What change ought he to give back if he is given a ten-dollar bill in payment?

8. Find the cost of a desk for $27.50, a chair for $9.75, a table for $12, a bookcase for $18.50, and a set of reference books for $67.80.

9. James bought a geography for \$1.15, an arithmetic for \$0.65, a grammar for \$0.48, a block of paper for \$0.08, and a pencil for \$0.03. How much less than \$3.00 did he pay for all?

DRILL IN MULTIPLICATION *Written*

$3 \times 4 \times 5 = ?$ $4 \times 5 \times 3 = ?$ $5 \times 3 \times 4 = ?$

The order in which numbers are multiplied together does not affect the product.

Multiply, selecting your multipliers so as to make your work as easy as possible:

1. $18 \times 50 \times 2$	**2.** $20 \times 24 \times 5$	**3.** $25 \times 45 \times 4$
4. $10 \times 36 \times 50$	**5.** $75 \times 26 \times 2$	**6.** $15 \times 19 \times 8$
7. $35 \times 15 \times 4$	**8.** $60 \times 57 \times 20$	**9.** $16 \times 32 \times 5$
10. 308×64	**11.** 876×75	**12.** 963×56
13. 3729×78	**14.** 5087×46	**15.** 7567×75
16. 436×208	**17.** 804×279	**18.** 225×306
19. 506×3468	**20.** 2387×207	**21.** 5682×256
22. 5468×357	**23.** 864×7678	**24.** 546×6807
25. 624×9034	**26.** 504×6327	**27.** 4657×406

28. At \$16.75 each, what will 5 gas stoves cost?

29. What must be paid for 14 hammocks at \$2.98 each?

30. A crate of berries contains 32 quart baskets. How many quarts in 5 lots of 12 crates each?

31. Mr. Howe bought 3 32-quart crates of strawberries at 12 cents a quart and sold them at 15 cents a quart. How much did he make?

32. Mr. Parker raised 17 bushels of pears. He sold 8 bushels at $1.05 a bushel, and the rest at $0.85 a bushel. How much did he receive for them?

33. After buying 6 head of cattle at $65 each, Mr. Turner had $27 left. How much money had he at first?

DRILL IN DIVISION *Written*

Divide, and test your work:

	A	*B*	*C*
1.	$801 ÷ 27	2765 ÷ 44	247,583 ÷ 64
2.	$765 ÷ 34	8327 ÷ 65	627,862 ÷ 75
3.	$896 ÷ 56	6754 ÷ 36	837,921 ÷ 29
4.	$27.95 ÷ 65	46,810 ÷ 84	247,583 ÷ 304
5.	$52.48 ÷ 82	67,632 ÷ 95	507,381 ÷ 409
6.	$34.08 ÷ 76	26,981 ÷ 43	729,843 ÷ 652
7.	$64.86 ÷ 138	48,366 ÷ 54	720,480 ÷ 432
8.	$133.92 ÷ 124	51,302 ÷ 208	837,641 ÷ 751
9.	$528.75 ÷ 225	64,730 ÷ 352	808,732 ÷ 364
10.	$739.44 ÷ 316	90,387 ÷ 525	976,068 ÷ 575

11. At 9 cents a yard Ella paid 45 cents for cloth. How many yards did she buy? (As many yards as 9 is contained times in 45.)

12. Esther paid 72 cents for 6 boxes of raisins. What was that a box? (One box cost ⅙ of 72 cents.)

13. At $6 a cord how many cords of wood can be bought for $912?

14. The grocer paid $702 for 54 barrels of sugar. What was the price per barrel?

15. A lot of land cost $6244. It was divided into 28 lots. What was each lot worth?

16. A stable keeper bought horses at $137 each. He paid $1096. How many did he buy?

17. Three lawn mowers were sold for $19.35. What was that apiece?

COMPARISON OF NUMBERS *Oral*

1. Compare 18 with 6. 18 is 3 times 6.

2. Compare 6 with 18. 6 is $\frac{1}{3}$ of 18.

Compare:

3. 10 with 2 4 with 20 30 with 6 5 with 40 27 with 9

4. 24 with 6 8 with 24 48 with 12 3 with 21 28 with 4

5. 56 with 7 9 with 36 54 with 9 7 with 63 32 with 8

NOTE. This exercise may be extended by comparing the second number in each couplet with the first.

6. A newsboy buys 5 papers for 3 cents. How many does he get for 15 cents?

HINT. Compare 15 cents with 3 cents.

7. Six boxes of raisins cost 75 cents. What will 2 boxes cost?

HINT. Compare 2 boxes with 6 boxes.

8. The grocer sells 4 pounds of sugar for a quarter. How many pounds does he sell for a dollar? For a dollar and a half?

9. Eight bars of soap weigh 36 pounds. What do 2 bars weigh?

10. Chester pays 25 cents for 8 oranges. Two dozen will cost ——.

11. Harriet buys 12 papers of needles for 20 cents. This is how many papers for 5 cents?

12. Mr. Perry pays $49.50 for 15 sheep. What will 5 more cost at the same rate?

13. For 2 cords of wood I paid $13.50. What will 10 cords cost?

DICTATION EXERCISES

1. $24 \div 3, \times 9, \div 12, \times 9, -5, \div 7, +20, \div 3 = ?$

2. $17 + 8, \times 2, +4, \div 6, +3, \times 7, +6, \div 10, \times 5 = ?$

3. $56 \div 8, \times 4, +2, \div 5, \times 7, +3, \div 5, -7, \times 5 = ?$

4. $42 \div 7, \times 9, +6, \div 5, +8, \div 2, -7, \times 9, +7 = ?$

5. $32 \div 4, +7, \div 5, \times 8, +4, \div 7, \times 16, -4, \div 6 = ?$

NOTE. For securing concentration of attention, this form of oral drill is unexcelled if used daily for a few moments. Numbers must be dictated rapidly to make exercise effective.

MISCELLANEOUS PROBLEMS *Written*

1. The following represents the cash receipts of a coal firm for one week:

Kind	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Totals
Furnace	$420.87	$473.19	$296.89	$318.54	$387.53	$464.59	$
Stove	384.60	297.64	372.23	376.53	455.90	278.83	$
No. 1 Nut	297.83	308.07	424.86	565.49	387.37	588.10	$
No. 2 Nut	378.69	420.00	375.50	482.96	300.87	249.50	$
Soft	684.17	367.29	294.73	783.59	462.82	539.42	$
Totals	$	$	$	$	$	$	$

(*a*) Find the amount received each day.

(*b*) Find the amount received for the week.

(*c*) Find the total receipts for each kind of coal for the week.

(*d*) Find the total receipts for all kinds during the week.

2. If 15 books cost $12.75, what is the cost of 1?

3. At $12.75 each, what will 15 plows cost?

4. At $0.75 a bushel, what is the value of the corn raised on 26 acres, if each acre produces 37 bushels?

5. A bushel of corn in the ear weighs 70 pounds. How many bushels are there in a car of 15,750 pounds?

6. How many times at $2 a time must the blacksmith shoe the farmer's horse to pay for 5 bushels of potatoes at $0.50 a bushel and 2 barrels of apples at $1.75 a barrel?

7. Lime absorbs $2\frac{1}{4}$ times its weight in water. How many pounds of water will be required to slake 6 casks of lime of 240 pounds each?

8. What will 30 acres of land cost at the rate of 6 acres for $336?

9. Mr. Clark buys of Mr. Hodge 3 acres of land at $84 an acre. Mr. Hodge buys of Mr. Clark 18 tons of hay at $16 a ton. In order to settle the account how much money must be paid, and who must pay it?

10. A farmer had $440. With $192 he bought 24 sheep. With the rest he bought 4 cows. What did each sheep cost? Each cow?

11. A coal dealer paid $900 for coal at $ 5 a ton. He sold it at $6.50 a ton. How much did he gain?

12. A fruit dealer bought 36 baskets of peaches for $30.60. He sold 27 baskets at $0.95 each and the rest at $1.15 each. How much did he gain?

13. By selling 42 acres of timber land for $2148 a man gained $804. What did the land cost him an acre?

14. The railway fare to a place 18 miles away is 54 cents. How far away is a place the fare to which is 72 cents?

15. On a lot costing $896 there was built a house costing $4\frac{1}{2}$ times as much. What was the cost of the entire property?

16. What is the cost of 6 cases of straw hats, each case containing 12 dozen, and each hat costing 79 cents?

17. Mr. Adams bought an automobile for $975, paying $450 in cash, and agreeing to pay the rest at $75 a month. How long did it take him to pay for it?

18. A 36-pound tub of butter was bought for $9.90 and retailed at 32 cents a pound. Did the dealer lose or gain? How much?

19. A cask of 84 gallons of molasses cost $37.80. Seven gallons leaked out and the rest was sold at 48 cents a gallon. Did the grocer gain or lose? How much?

20. The pupils of the Adams school spent $10.65 for their school garden. They bought 9 dozen bulbs at 35 cents a dozen and 15 shrubs. How much did each shrub cost?

FACTORS *Oral*

When two or more numbers are multiplied together, the result is a *product.*

The numbers multiplied together are the *factors* of the product. Thus, 3 and 5 are the factors of their product, 15. 2, 3, and 5 are the factors of 30.

Any product is exactly divisible by any of its factors.

Find the missing factors:

1. —— × 9 = 54 9 × —— = 63 —— × 6 = 72
6 × —— = 42

2. —— × 5 = 30 7 × —— = 56 —— × 7 = 35
4 × —— = 32

3. —— × 7 = 63 3 × —— = 36 —— × 9 = 72
8 × —— = 96

4. —— × 12 = 84 6 × —— = 54 —— × 12 = 144
12 × —— = 132

The process of separating a number into its factors is *factoring.*

Separate into two factors:

5. 14 22 33 45 81 42 70 63 66 35

6. 56 64 21 32 72 84 54 96 55 108

7. Separate 24 into as many groups of two factors as you can. Thus, 2 × 12, 3 × 8, 4 × 6.

Name all the groups of two factors that make:

8. 16 28 20 40 50 80 72 90 84 42

9. 18 30 48 60 32 96 36 64 90 100

Separate each of these numbers into three factors:

10.	12	18	27	30	28	50	63	45	70	100
11.	32	40	66	48	20	72	54	60	56	144

Name the two equal factors of:

12.	4	9	25	49	81	64	144
13.	100	900	2500	4900	8100	3600	400

NOTE. Every number, of course, may be said to be made up of two factors consisting of itself and 1, but in giving the factors of a number the number itself and 1 are not generally included.

Numbers that can be separated into factors are *composite* numbers.

14. Name the composite numbers below 20.

Numbers that cannot be separated into factors are *prime* numbers.

15. Name the prime numbers below 20.

A *prime factor* is a prime number used as a factor.

16. What are the prime factors of 72?

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

Dividing 72 by the prime number 2, we get 36; dividing 36 by 2, we get 18; dividing 18 by 2, we get 9; dividing 9 by the prime number 3, we get 3. All these divisors and the last quotient are prime numbers, and their product is 72. $2 \times 2 \times 2 \times 3 \times 3 = 72$. Therefore, the prime factors of 72 are 2, 2, 2, 3, and 3.

NOTE. The above example is inserted for illustration. The method given may be used if necessary, but pupils should be taught to find prime factors by inspection whenever possible. Thus, 72 may be thought of as 8×9; then 8 may be thought of as $2 \times 2 \times 2$, and 9 as 3×3.

Name the prime factors of:

17. 18 20 24 30 32 36 45 48 60 56

18. 84 50 66 80 90 64 81 63 54 100

GREATEST COMMON DIVISOR *Oral*

A number that will exactly divide a given number is an *exact divisor.*

1. Name a number that will exactly divide both 6 and 9; 8 and 12; 10 and 15; 12 and 18.

A number that will exactly divide two or more numbers is a *common divisor.*

2. Name the greatest number that will exactly divide 12 and 16; 18 and 24; 24 and 32; 30 and 40.

The greatest number that will exactly divide two or more numbers is their *greatest common divisor* (*g. c. d.*).

The greatest common divisor of two or more numbers is often called their greatest common factor.

Name the greatest common divisor of:

3. 16, 20	**4.** 22, 33	**5.** 18, 27	**6.** 27, 36
7. 14, 35	**8.** 32, 40	**9.** 11, 15	**10.** 36, 48
11. 35, 42	**12.** 20, 35	**13.** 28, 42	**14.** 63, 72
15. 56, 63	**16.** 45, 54	**17.** 28, 49	**18.** 24, 32
19. 6, 9, 12	**20.** 8, 12, 20	**21.** 12, 15, 18	**22.** 10, 15, 25
23. 15, 18, 30	**24.** 18, 24, 30	**25.** 12, 15, 21	**26.** 21, 28, 35
27. 18, 27, 45	**28.** 22, 33, 55	**29.** 24, 32, 40	**30.** 24, 36, 60

LEAST COMMON MULTIPLE *Oral*

When two or more whole numbers are multiplied together, their product is a *multiple* of each of the numbers. Thus, 15 is a multiple of both 3 and 5.

Any multiple of a number is exactly divisible by the number.

1. Name all the factors whose product is 12. Thus,

$$2 \times 6, \quad 3 \times 4, \quad 2 \times 2 \times 3.$$

12 is a *common multiple* of 2, 3, 4, and 6, and is exactly divisible by each of them.

24, 36, 48, 60 are also common multiples of 2, 3, 4, and 6.

As 12 is the *least multiple* that contains 2, 3, 4, and 6, it is their *least common multiple* (*l. c.m*).

The least common multiple of two or more numbers is the least number that is exactly divisible by each of the numbers.

What is the least common multiple of 5 and 6? Of 4 and 6? Of 3 and 9?

Find the least common multiple of:

2. 4 and 8	**3.** 7 and 8	**4.** 6 and 8	**5.** 6 and 9
6. 8 and 9	**7.** 8 and 12	**8.** 4 and 10	**9.** 6 and 10
10. 9 and 12	**11.** 6 and 15	**12.** 5 and 15	**13.** 8 and 24
14. 2, 4, 8	**15.** 4, 8, 16	**16.** 2, 3, 4	**17.** 3, 4, 6
18. 2, 4, 5	**19.** 3, 6, 9	**20.** 4, 5, 10	**21.** 2, 5, 20
22. 3, 6, 5	**23.** 4, 9, 36	**24.** 4, 5, 6	**25.** 3, 4, 9
26. 4, 6, 8	**27.** 6, 9, 12	**28.** 4, 8, 12	**29.** 3, 4, 5

CANCELLATION *Oral and Written*

$60 \div 20 = 6 \times 10$ divided by 2×10.

What common factor is found in both dividend and divisor?

By taking out the common factor 10 from both dividend and divisor, is the quotient changed?

What is the quotient of $60 \div 20$? Of $6 \div 2$?

Dividing both dividend and divisor by the same number does not affect the quotient.

Tell what common factors may be taken out of, or canceled from, both dividend and divisor:

1. 12×3 divided by 5×3
2. 10×3 divided by 10×2
3. 10×8 divided by 3×8
4. 21×7 divided by 4×7
5. 11×5 divided by 11×3
6. 11×12 divided by 12×3

7. In the expression 12×10 divided by 8×3 what common factors will divide both dividend and divisor? What in 14×10 divided by 5×7?

The process of dividing both dividend and divisor by the same number, or of striking out factors common to both dividend and divisor, is *cancellation.*

8. Divide 16×35 by 4×7.

$$\frac{\overset{4}{\cancel{16}} \times \overset{5}{\cancel{35}}}{\underset{1}{\cancel{4}} \times \underset{1}{\cancel{7}}} = \frac{20}{1} = 20$$

Write the dividend above a line and the divisor below it. Divide the 16 in the dividend and the 4 in the divisor by the common factor 4, writing the quotient 4 over the 16, and the quotient 1 under the 4. In like manner divide both dividend and divisor by the common factor 7.

The factors remaining in the dividend are 4 and 5, and their product is 20. The factors remaining in the divisor are 1 and 1, and their product is 1. $\frac{20}{1} = 20$.

In practice we do not write the 1's. We always remember, however, that when a factor is canceled 1 is understood to take its place.

9. Divide 56×18 by 8×9.

10. What is the quotient of 42×10 divided by 7×5?

11. How many times is 4×3 contained in 6×8?

Find quotients:

12. $\frac{4 \times 12}{2 \times 6}$ 13. $\frac{6 \times 25}{3 \times 5}$ 14. $\frac{20 \times 30}{15 \times 10}$ 15. $\frac{18 \times 30}{6 \times 5}$

16. $\frac{27 \times 18}{9 \times 3}$ 17. $\frac{28 \times 35}{4 \times 7}$ 18. $\frac{15 \times 50}{5 \times 5 \times 5}$ 19. $\frac{60 \times 30}{5 \times 12}$

20. $(22 \times 18) \div (11 \times 6)$ 21. $(35 \times 42) \div (14 \times 7)$

22. $(35 \times 42) \div (49 \times 6)$ 23. $(63 \times 72) \div (24 \times 21)$

24. $(33 \times 48) \div (12 \times 22)$ 25. $(54 \times 54) \div (6 \times 18)$

26. $(60 \times 27) \div (18 \times 45)$ 27. $(35 \times 84) \div (49 \times 30)$

Divide:

28. $\frac{6 \times 10 \times 15}{25 \times 2 \times 2}$ 29. $\frac{12 \times 15 \times 24}{20 \times 4 \times 18}$ 30. $\frac{9 \times 8 \times 10}{30 \times 2 \times 3}$

31. $\frac{18 \times 30 \times 22}{33 \times 10 \times 9}$ 32. $\frac{50 \times 42 \times 20}{35 \times 25 \times 12}$ 33. $\frac{36 \times 45 \times 27}{18 \times 15 \times 9}$

34. $\frac{11 \times 30 \times 28}{15 \times 22 \times 7}$ 35. $\frac{60 \times 42 \times 54}{9 \times 20 \times 6}$ 36. $\frac{44 \times 56 \times 96}{48 \times 77 \times 16}$

FRACTIONS

A unit is a single thing; as 1 apple.

A fraction is one or more of the equal parts of a unit; as $\frac{3}{4}$ of an apple.

$\frac{3}{4}$ of an apple means that an apple has been divided into 4 equal parts and 3 of these parts taken.

1. In the expression $\frac{3}{8}$ of a yard, what figure shows the number of equal parts into which the unit is divided?

The figure below the line is the *denominator;* it denominates or names the number of parts into which the unit is divided; it is the *namer.*

2. In the expression $\frac{3}{8}$ of a yard, what figure shows the number of parts taken?

The figure above the line is the *numerator;* it numerates or tells the number of parts taken; it is the *numberer.*

The numerator and the denominator are the *terms* of the fraction.

3. Read these fractions and tell what the terms of each fraction show: $\frac{5}{8}$; $\frac{6}{7}$; $\frac{4}{5}$; $\frac{9}{10}$; $\frac{17}{20}$.

4. Write in figures and tell what each fraction means: five sixths; eight ninths; eleven twelfths; thirteen twenty-firsts.

5. Write an expression which will show that something has been divided into nine equal parts and four of those parts taken.

6. Explain $\frac{4}{5}$ of a mile; $\$\frac{4}{5}$; $\frac{3}{8}$ bu.; $\frac{3}{4}$ gal.

A unit may also be regarded as a group of things treated as a single thing. Thus, $\frac{3}{4}$ of a dozen oranges means that 12 oranges have been separated into 4 equal groups of 3 oranges each, and that 3 of these groups, or 9 oranges, have been taken.

In studying fractions remember:

First. The only difference between an integer, or whole number, and a fraction is that an integer is a whole thing, while a fraction is part of the whole thing.

Second. The denominator of a fraction simply tells with what kind of things we are dealing; that is, it simply gives a name to the fraction.

Third. The numerator simply tells the number of parts taken.

Fourth. A fraction must always be treated as if it were a whole number.

A *proper fraction* is a fraction whose numerator is less than its denominator; as $\frac{7}{8}$, $\frac{5}{7}$, $\frac{11}{12}$.

An *improper fraction* is a fraction whose numerator is equal to or greater than its denominator; as $\frac{5}{5}$, $\frac{8}{8}$, $\frac{16}{5}$, $\frac{7}{3}$.

7. Write a proper fraction whose denominator is 5; 8; 12; 10; 3.

8. Write a proper fraction whose numerator is 3; 7; 9; 4; 10.

9. Write an improper fraction whose numerator is 7; 6; 4; 3; 5.

10. Write an improper fraction whose denominator is 3; 6; 8; 9; 10.

A *mixed number* is a whole number and a fraction united; as $2\frac{1}{2}$, $3\frac{3}{4}$, $4\frac{5}{9}$.

CHANGING THE FORM OF FRACTIONS

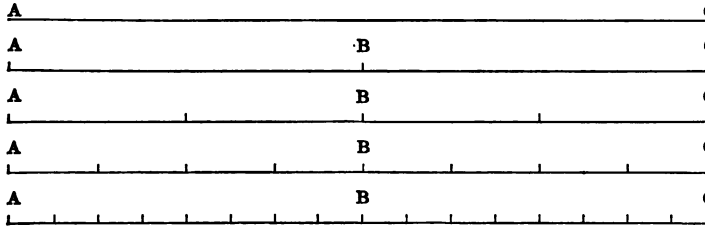
1. Cut from paper a strip 1 inch wide and 12 inches long. Place the ends together and fold into two equal parts. Show that $1 = \frac{2}{2}$.

2. Fold again and crease into four equal parts. Show that $1 = \frac{4}{4}$. Show that $\frac{1}{2} = \frac{2}{4}$.

3. Fold and crease into eight equal parts. 1 = how many eighths? $\frac{1}{2}$ = how many eighths? $\frac{1}{4}$ = how many eighths?

4. Fold another strip into two equal parts. Fold this double strip into three equal parts. 1 = how many sixths? 1 = how many thirds? $\frac{1}{2}$ = how many sixths? $\frac{1}{3}$ = how many sixths? $\frac{2}{3}$ = how many sixths?

To the Teacher: Simple fractions and their equivalents may be shown in this or some other simple manner. The extent to which such work is carried must be determined by the needs of individual pupils. While objective presentation should be used freely, care should be taken not to make pupils dependent on its use. That which is at first a help may easily become a hindrance to progress.



If the line AC be divided into two equal parts, AB is $\frac{1}{2}$ of AC; if divided into four equal parts, AB is $\frac{2}{4}$; if divided into eight equal parts, AB is $\frac{4}{8}$; if divided into sixteen equal parts, AB is $\frac{8}{16}$; that is, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{8}$, and $\frac{8}{16}$ of the line AC are of equal value, and represent the same thing — the line AB.

Notice, that in changing $\frac{1}{2}$ to $\frac{2}{4}$ we have twice as many parts in the line AC, and also twice as many parts in the line AB. In changing $\frac{1}{2}$ to $\frac{4}{8}$, we have four times as many parts in the line AC, and also four times as many parts in the line AB. In changing $\frac{1}{2}$ to $\frac{8}{16}$, we have eight times

as many parts in the line AC, and eight times as many parts in the line AB.

$$\frac{1\times 2}{2\times 2}=\frac{2}{4} \qquad \frac{1\times 4}{2\times 4}=\frac{4}{8} \qquad \frac{1\times 8}{2\times 8}=\frac{8}{16}$$

In changing $\frac{8}{16}$ to $\frac{4}{8}$, we have one half as many parts in in the line AC, and one half as many parts in the line AB. In changing $\frac{8}{16}$ to $\frac{2}{4}$, we have one fourth as many parts in the line AC, and one fourth as many parts in the line AB. In changing $\frac{8}{16}$ to $\frac{1}{2}$, we have one eighth as many parts in the line AC, and one eighth as many parts in the line AB.

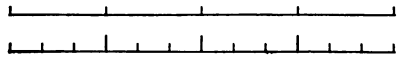
$$\frac{8\div 2}{16\div 2}=\frac{4}{8} \qquad \frac{8\div 4}{16\div 4}=\frac{2}{4} \qquad \frac{8\div 8}{16\div 8}=\frac{1}{2}$$

From this exercise we learn that

Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.

CHANGING TO HIGHER TERMS *Oral*

1. Change $\frac{3}{4}$ to twelfths.



$$\frac{3\times 3}{4\times 3}=\frac{9}{12}$$

The fraction $\frac{3}{4}$ shows that the unit has been separated into 4 equal parts and 3 of those parts taken. If we separate the unit into twelfths, or three times as many parts, we have three times as many parts for the numerator. $\frac{1}{4}=\frac{3}{12}$; $\frac{3}{4}=\frac{9}{12}$.

To change a fraction to higher terms, we multiply both terms of the fraction by that number which will give the required denominator.

2. Why must we multiply both terms of the fraction by the same number?

Change:

3. To fourths: $\frac{1}{2}$

4. To sixths: $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$

5. To eighths: $\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$

6. To ninths: $\frac{1}{3}$ $\frac{2}{3}$

7. To tenths: $\frac{1}{2}$ $\frac{1}{5}$ $\frac{2}{5}$

8. To twelfths: $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{1}{6}$ $\frac{5}{6}$

9. To fifteenths: $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{5}$ $\frac{3}{5}$ $\frac{4}{5}$

10. To sixteenths: $\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{3}{8}$ $\frac{5}{8}$

11. To eighteenths: $\frac{1}{2}$ $\frac{2}{3}$ $\frac{5}{6}$ $\frac{1}{9}$ $\frac{5}{9}$

12. To twentieths: $\frac{1}{2}$ $\frac{3}{4}$ $\frac{2}{5}$ $\frac{4}{5}$ $\frac{7}{10}$

CHANGING TO LOWER TERMS *Oral and Written*

1. Change $\frac{6}{9}$ to thirds.

$\frac{6 \div 3}{9 \div 3} = \frac{2}{3}$

This means change $\frac{6}{9}$ to a fraction with 3 for its denominator.

In order to get 3 for a denominator, we divide 9 by 3. In order not to change the value of the fraction, we must also divide the numerator by 3.

NOTE. If necessary, let pupils show by folding paper or by diagram that $\frac{6}{9} = \frac{2}{3}$.

2. Change to 4ths: $\frac{2}{8}$ $\frac{3}{12}$ $\frac{9}{12}$ $\frac{4}{16}$ $\frac{12}{16}$ $\frac{5}{20}$

3. Change to 5ths: $\frac{2}{10}$ $\frac{4}{10}$ $\frac{8}{10}$ $\frac{4}{20}$ $\frac{12}{20}$ $\frac{6}{30}$

4. Change to 6ths: $\frac{2}{12}$ $\frac{10}{12}$ $\frac{3}{18}$ $\frac{15}{18}$ $\frac{4}{24}$ $\frac{5}{30}$

5. Change to 9ths: $\frac{2}{18}$ $\frac{4}{18}$ $\frac{8}{18}$ $\frac{10}{18}$ $\frac{3}{27}$ $\frac{4}{36}$

6. Change to 12ths: $\frac{2}{24}$ $\frac{10}{24}$ $\frac{14}{24}$ $\frac{8}{36}$ $\frac{4}{48}$ $\frac{5}{60}$

7. Change $\frac{15}{20}$ to its simplest form.

$\frac{15 \div 5}{20 \div 5} = \frac{3}{4}$

Since the factor 5 is common to both numerator and denominator, we can divide both terms by 5 without changing the value of the fraction. As the numerator and denominator of the fraction $\frac{3}{4}$ have no common factor, the fraction $\frac{15}{20}$ has been changed to its simplest form, or, as we say, to its lowest terms.

A fraction is in its lowest terms when its numerator and denominator have no common factor.

Change to lowest terms:

8. $\frac{6}{9}$ $\frac{8}{10}$ $\frac{10}{16}$ $\frac{6}{14}$ $\frac{12}{18}$ $\frac{15}{21}$ $\frac{14}{24}$ $\frac{20}{20}$

9. $\frac{8}{12}$ $\frac{6}{15}$ $\frac{6}{6}$ $\frac{12}{16}$ $\frac{8}{18}$ $\frac{6}{20}$ $\frac{14}{21}$ $\frac{6}{24}$

10. $\frac{6}{12}$ $\frac{8}{14}$ $\frac{9}{12}$ $\frac{12}{15}$ $\frac{8}{20}$ $\frac{6}{21}$ $\frac{2}{28}$ $\frac{15}{24}$

11. $\frac{5}{15}$ $\frac{12}{21}$ $\frac{12}{20}$ $\frac{18}{21}$ $\frac{9}{24}$ $\frac{16}{16}$ $\frac{14}{22}$ $\frac{18}{27}$

12. $\frac{10}{18}$ $\frac{9}{15}$ $\frac{10}{12}$ $\frac{16}{20}$ $\frac{14}{16}$ $\frac{9}{21}$ $\frac{10}{24}$ $\frac{19}{19}$

13. $\frac{30}{42}$

$\frac{30 \div 2 = 15 \div 3 = 5}{42 \div 2 = 21 \div 3 = 7}$

Or

$\frac{30 \div 6 = 5}{42 \div 6 = 7}$

Dividing both terms of $\frac{30}{42}$ by 2, we get $\frac{15}{21}$; dividing both terms of $\frac{15}{21}$ by 3, we get $\frac{5}{7}$.

We can change this fraction more quickly by dividing both terms by their greatest common factor, 6.

14. $\frac{18}{42}$ 15. $\frac{49}{63}$ 16. $\frac{48}{78}$ 17. $\frac{48}{84}$ 18. $\frac{75}{135}$

19. $\frac{30}{48}$ 20. $\frac{42}{70}$ 21. $\frac{60}{72}$ 22. $\frac{84}{144}$ 23. $\frac{60}{150}$

24. $\frac{24}{54}$ 25. $\frac{56}{63}$ 26. $\frac{36}{80}$ 27. $\frac{90}{120}$ 28. $\frac{96}{108}$

29. $\frac{42}{63}$ 30. $\frac{42}{54}$ 31. $\frac{63}{84}$ 32. $\frac{48}{96}$ 33. $\frac{105}{150}$

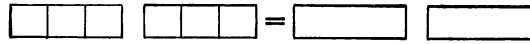
34. $\frac{30}{72}$ 35. $\frac{72}{84}$ 36. $\frac{70}{98}$ 37. $\frac{90}{108}$ 38. $\frac{125}{175}$

To change a fraction to its lowest terms, we cancel the factors common to both numerator and denominator; or we divide both terms by their greatest common factor.

NOTE. Do not now require pupils to give rules or technical explanations of process. The main thing at present is to see that pupils understand and apply the principles.

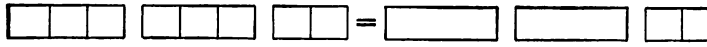
CHANGING IMPROPER FRACTIONS TO WHOLE OR MIXED NUMBERS *Oral and Written*

1. Change $\frac{6}{3}$ to a whole number.



$\frac{6}{3} = 2$ Since there are 3 *thirds* ($\frac{1}{3}$) in 1 unit, in 6 *thirds* ($\frac{6}{3}$) there are as many units as there are 3's in 6; that is, 2 units.

2. Change $\frac{8}{3}$ to a mixed number.



$\frac{8}{3} = 2\frac{2}{3}$ Since there are 3 thirds in 1 unit, in 8 thirds there are as many units as there are 3's in 8; that is, 2 units and 2 thirds of a unit.

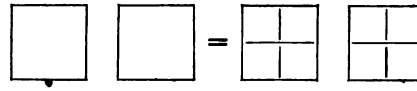
To change an improper fraction to a whole or mixed number, we divide the numerator by the denominator.

Change to whole or mixed numbers:

3. $\frac{24}{7}$	**4.** $\frac{36}{9}$	**5.** $\frac{15}{8}$	**6.** $\frac{42}{5}$	**7.** $\frac{56}{7}$
8. $\frac{27}{4}$	**9.** $\frac{28}{9}$	**10.** $\frac{24}{8}$	**11.** $\frac{48}{8}$	**12.** $\frac{29}{6}$
13. $\frac{43}{8}$	**14.** $\frac{36}{9}$	**15.** $\frac{40}{9}$	**16.** $\frac{25}{7}$	**17.** $\frac{30}{5}$
18. $\frac{15}{2}$ in.	**19.** $\frac{17}{4}$ ft.	**20.** $\frac{28}{4}$ yd.	**21.** $\frac{19}{2}$ ft.	**22.** $\frac{11}{2}$ qt.
23. $\frac{35}{4}$ gal.	**24.** $\frac{35}{8}$ pk.	**25.** $\frac{44}{4}$ bu.	**26.** $\$\frac{5}{5}$	**27.** $\$\frac{20}{4}$

CHANGING WHOLE OR MIXED NUMBERS TO IMPROPER FRACTIONS *Oral and Written*

1. Change 2 to fourths.



$2 = \frac{2}{1}$

$\frac{2}{1} \times \frac{4}{4} = \frac{8}{4}$

Since there are 4 *fourths* in one unit, in 2 units there are 2 times 4 *fourths*, or 8 *fourths*.

To change a whole number to an improper fraction, we multiply the whole number by the required denominator, and write the product over the required denominator.

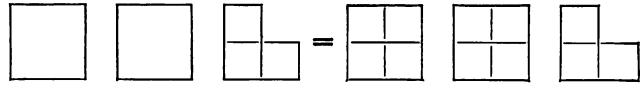
2. Change to halves: 1 2 3 4 5

3. Change to thirds: 1 2 4 6 9

4. How many fifths are there in 1? 3? 6? 7? 8?

5. Express as fractions with 8 for a denominator: 3 5 7 8 10

6. Change $2\frac{3}{4}$ to fourths.



$2\frac{3}{4} = \frac{11}{4}$

Two units equal 8 *fourths*; 8 *fourths* and 3 *fourths* are 11 *fourths*.

To change a mixed number to an improper fraction, we multiply the whole number by the denominator of the fraction, to the product add the numerator, and write the sum over the denominator.

Write as improper fractions:

7. $4\frac{2}{5}$	8. $7\frac{3}{4}$	9. $5\frac{3}{8}$	10. $2\frac{5}{9}$
11. $4\frac{1}{6}$	12. $8\frac{2}{3}$	13. $3\frac{5}{7}$	14. $5\frac{2}{5}$
15. $2\frac{5}{6}$	16. $3\frac{1}{10}$	17. $3\frac{8}{9}$	18. $5\frac{7}{8}$
19. $4\frac{9}{10}$	20. $4\frac{2}{7}$	21. $6\frac{3}{5}$	22. $3\frac{1}{8}$
23. $4\frac{2}{3}$ ft.	24. $1\frac{1}{2}$ pt.	25. $3\frac{1}{2}$ qt.	26. $5\frac{3}{4}$ in.
27. $\$4\frac{2}{5}$	28. $7\frac{5}{8}$ pk.	29. $9\frac{3}{4}$ bu.	30. $5\frac{5}{6}$ mi.

REVIEW EXERCISE *Written*

1. Write a proper fraction using 5 and 3 for its terms.

2. Change the form of the fraction you have written without changing its value.

3. Change $\frac{2}{3}$ to ninths; $\frac{3}{4}$ to 12ths; $\frac{3}{8}$ to 16ths.

4. In the fraction $\frac{9}{12}$, what factor is common to both terms? To what simpler fraction is $\frac{9}{12}$ equal?

5. Take out the common factors in these fractions: $\frac{4}{20}$ $\frac{15}{18}$ $\frac{10}{16}$ $\frac{10}{24}$ $\frac{5}{30}$.

6. Take out all the common factors in $\frac{30}{48}$.

7. Change to lowest terms: $\frac{12}{16}$ $\frac{24}{30}$ $\frac{36}{60}$ $\frac{42}{63}$ $\frac{72}{84}$.

8. Write an improper fraction whose terms are 12 and 3. Change it to a whole number.

9. Write two improper fractions that can be changed to mixed numbers.

10. What is a mixed number?

11. Write five mixed numbers and change them to improper fractions.

12. Change 3 to halves; 6 to fifths; 5 to eighths; 4 to twelfths.

ADDITION OF FRACTIONS *Oral and Written*

1. Add $\frac{3}{7}$ and $\frac{2}{7}$.

$$\frac{3}{\text{apples}} + \frac{2}{\text{apples}} = \frac{5}{\text{apples}}$$

$$\frac{3}{\text{sevenths}} + \frac{2}{\text{sevenths}} = \frac{5}{\text{sevenths}}$$

$$\tfrac{3}{7} + \tfrac{2}{7} = \tfrac{5}{7}$$

The denominator names the fraction; it simply tells the kind of things with which we are dealing.

Add:

2. $\frac{1}{6} + \frac{1}{6} + \frac{3}{6}$ **3.** $\frac{1}{5} + \frac{1}{5} + \frac{2}{5}$ **4.** $\frac{3}{8} + \frac{2}{8} + \frac{1}{8}$

5. $\frac{2}{9} + \frac{3}{9} + \frac{2}{9}$ **6.** $\frac{1}{11} + \frac{4}{11} + \frac{5}{11}$ **7.** $\frac{5}{12} + \frac{1}{12} + \frac{1}{12}$

8. $\frac{3}{16} + \frac{1}{16} + \frac{5}{16}$ **9.** $\frac{3}{20} + \frac{7}{20} + \frac{9}{20}$ **10.** $\frac{4}{25} + \frac{3}{25} + \frac{11}{25}$

11. Add $\frac{5}{8}$ and $\frac{3}{4}$.

$\frac{5}{8} + \frac{3}{4} = ?$

$\frac{3}{4} = \frac{6}{8}$

$\frac{5}{8} + \frac{6}{8} = 1\frac{1}{8} = 1\frac{3}{8}$

$$\frac{5}{\text{quarts}} + \frac{3}{\text{pecks}} = ?$$

Since these quantities do not represent things of the same kind, they cannot be added. But, since 1 peck is equal to 8 quarts, 3 pecks may be expressed as 24 quarts. $\frac{5}{\text{quarts}} + \frac{24}{\text{quarts}} = \frac{29}{\text{quarts}}$.

Similarly, $\frac{5}{8} + \frac{3}{4}$. Since eighths and fourths represent unlike things, we cannot add them until we express them as like things; that is, as fractions having the same denominator, which we call a common denominator. The common denominator is 8. $\frac{3}{4} = \frac{6}{8}$.

12. Add $\frac{1}{4}$ and $\frac{1}{6}$.

c. d. = 12

$\frac{1}{4} = \frac{3}{12}$

$\frac{1}{6} = \frac{2}{12}$

$\frac{5}{12}$

We can express these fractions as 24ths, for 24 is a multiple of both 4 and 6. 12 is also a multiple of 4 and 6, and is the least multiple common to both. It simplifies our work to use the least common multiple of the denominators for the common denominator.

To add fractions, we express the fractions as equivalent ıctions having a common denominator, and write the sum the numerators over the common denominator.

NOTE. As much as possible of the work in fractions, both abstract l concrete, should be done orally.

Add, changing the fraction in the answer to its lowest ms:

3. $\frac{1}{2}+\frac{1}{6}$	14. $\frac{1}{2}+\frac{3}{8}$	15. $\frac{2}{3}+\frac{5}{6}$	16. $\frac{3}{4}+\frac{5}{8}$
7. $\frac{7}{10}+\frac{1}{2}$	18. $\frac{5}{12}+\frac{1}{2}$	19. $\frac{1}{2}+\frac{5}{16}$	20. $\frac{2}{3}+\frac{1}{12}$
L. $\frac{3}{4}+\frac{5}{12}$	22. $\frac{2}{15}+\frac{1}{3}$	23. $\frac{3}{5}+\frac{2}{15}$	24. $\frac{3}{4}+\frac{9}{16}$
5. $\frac{2}{5}+\frac{7}{10}$	26. $\frac{5}{6}+\frac{5}{12}$	27. $\frac{3}{8}+\frac{7}{16}$	28. $\frac{3}{4}+\frac{7}{12}$
9. $\frac{3}{10}+\frac{4}{5}$	30. $\frac{2}{5}+\frac{7}{15}$	31. $\frac{3}{4}+\frac{7}{16}$	32. $\frac{5}{6}+\frac{7}{12}$

Find sum of:

3. $\frac{1}{2}+\frac{1}{3}$	34. $\frac{1}{4}+\frac{1}{3}$	35. $\frac{1}{2}+\frac{2}{5}$	36. $\frac{2}{3}+\frac{2}{5}$
7. $\frac{3}{7}+\frac{1}{2}$	38. $\frac{1}{2}+\frac{2}{9}$	39. $\frac{2}{5}+\frac{1}{2}$	40. $\frac{2}{3}+\frac{3}{4}$
L. $\frac{1}{3}+\frac{4}{5}$	42. $\frac{1}{2}+\frac{5}{7}$	43. $\frac{2}{3}+\frac{4}{5}$	44. $\frac{2}{3}+\frac{1}{2}$
5. $\frac{4}{5}+\frac{1}{2}$	46. $\frac{2}{3}+\frac{3}{5}$	47. $\frac{4}{5}+\frac{1}{2}$	48. $\frac{5}{6}+\frac{1}{4}$
9. $\frac{3}{4}+\frac{5}{6}$	50. $\frac{1}{3}+\frac{3}{4}$	51. $\frac{1}{2}+\frac{1}{7}$	52. $\frac{1}{3}+\frac{1}{5}$

PROBLEMS

.. Mr. Smith has $\frac{1}{2}$ of an acre in one lot and $\frac{5}{8}$ of an e in another lot. How many acres are there in both ?

1. Miriam's spelling book cost $\frac{1}{5}$ of a dollar and her thmetic $\frac{1}{2}$ of a dollar. What part of a dollar did both t?

3. A cook used $\frac{7}{8}$ of a ton of coal in January and $\frac{3}{4}$ of a ton in February. How much did she use in both months?

4. Maggie bought $\frac{5}{6}$ of a yard of lace for an apron, and $\frac{2}{3}$ of a yard for a waist. How much lace did she buy?

5. A spelling lesson takes $\frac{1}{5}$ of an hour, and a reading lesson $\frac{1}{3}$ of an hour. What part of an hour is taken for both lessons?

To the Teacher: Many simple oral problems illustrating the principle under consideration should be given by the teacher. As far as possible, the problem material should be within the realm of the pupils' interest and experience. Local conditions will determine the character and content of problem work.

Pupils should be encouraged and required to make original problems based on their observation of the affairs of everyday life.

ADDITION

Written

Find the sum of:

1. $\frac{1}{3} + \frac{3}{4} + \frac{1}{6}$

$c.d. = 12$

$\frac{1}{3} = \frac{4}{12}$

$\frac{3}{4} = \frac{9}{12}$

$\frac{1}{6} = \frac{2}{12}$

$\frac{15}{12} = 1\frac{3}{12} = 1\frac{1}{4}$

2. $\frac{5}{8} + \frac{1}{2} + \frac{3}{4}$

3. $\frac{1}{2} + \frac{1}{6} + \frac{3}{4}$

4. $\frac{3}{10} + \frac{2}{5} + \frac{1}{2}$

5. $\frac{2}{3} + \frac{5}{6} + \frac{5}{6}$

6. $\frac{5}{6} + \frac{3}{4} + \frac{7}{12}$

7. $\frac{1}{6} + \frac{1}{2} + \frac{1}{3}$

8. $\frac{2}{3} + \frac{1}{2} + \frac{5}{6}$

9. $\frac{7}{15} + \frac{2}{3} + \frac{3}{5}$

10. $\frac{3}{5} + \frac{3}{10} + \frac{1}{2}$

11. $\frac{1}{2} + \frac{4}{7} + \frac{3}{14}$

12. $\frac{1}{4} + \frac{2}{3} + \frac{1}{2}$

13. $\frac{1}{4} + \frac{5}{6} + \frac{2}{3}$

14. $\$\frac{1}{2} + \$\frac{3}{5} + \$\frac{1}{10}$

15. $\frac{3}{4}$ gal. + $\frac{5}{6}$ gal. + $\frac{1}{2}$ gal.

16. $\frac{1}{6}$ yr. + $\frac{2}{3}$ yr. + $\frac{7}{12}$ yr.

17. $\frac{1}{3}$ yd. + $\frac{3}{4}$ yd. + $\frac{5}{12}$ yd.

18. $\frac{1}{2}$ bu. + $\frac{3}{8}$ bu. + $\frac{1}{4}$ bu.

19. $\frac{2}{3}$ mi. + $\frac{3}{4}$ mi. + $\frac{1}{12}$ mi.

PROBLEMS

1. John spent $\frac{1}{6}$ of his money for candy, $\frac{1}{8}$ for a ball, and $\frac{1}{4}$ for a bat. What part of his money did he spend?

2. Mary earned $\frac{2}{5}$ of a dollar, $\frac{1}{2}$ of a dollar, and $\frac{3}{10}$ of a dollar. How much did she earn in all?

3. Mr. Wright has $\frac{1}{16}$ of an acre of corn, $\frac{1}{8}$ of an acre of potatoes, and $\frac{1}{4}$ of an acre of onions. What part of an acre is used for all?

4. A bag of flour cost $\frac{7}{10}$ of a dollar, a bushel of potatoes $\frac{1}{2}$ of a dollar, and a peck of apples $\frac{1}{5}$ of a dollar. How much did all cost?

5. Mrs. Whiting paid $\$\frac{1}{5}$ for oranges, $\$\frac{1}{5}$ for sugar, and $\$\frac{1}{2}$ for peaches. What part of a dollar did she pay for all?

ADDITION OF MIXED NUMBERS *Written*

1. Add $9\frac{2}{3}$ and $6\frac{3}{4}$

$c.d. = 12$

$9\frac{2}{3} = 9\frac{8}{12}$

$6\frac{3}{4} = 6\frac{9}{12}$

$16\frac{5}{12}$

Express both fractions as 12ths.
$\frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1\frac{5}{12}$. Write $\frac{5}{12}$ under the fractions and add 1 to the sum of the whole numbers.

Add:

2. $3\frac{3}{4}+4\frac{1}{2}$	3. $2\frac{1}{2}+7\frac{5}{6}$	4. $3\frac{5}{8}+4\frac{1}{2}$	5. $6\frac{11}{16}+7\frac{1}{2}$
6. $4\frac{2}{3}+2\frac{5}{6}$	7. $5\frac{1}{6}+8\frac{1}{2}$	8. $4\frac{5}{8}+9\frac{3}{4}$	9. $5\frac{9}{16}+3\frac{7}{8}$
10. $7\frac{1}{2}+3\frac{7}{10}$	11. $5\frac{3}{10}+3\frac{4}{5}$	12. $7\frac{11}{16}+2\frac{3}{4}$	13. $4\frac{2}{3}+3\frac{5}{9}$
14. $7\frac{5}{6}+8\frac{7}{12}$	15. $3\frac{3}{4}+5\frac{7}{12}$	16. $8\frac{5}{12}+6\frac{2}{3}$	17. $5\frac{1}{2}+8\frac{1}{12}$
18. $8\frac{2}{3}+8\frac{1}{2}$	19. $5\frac{1}{2}+2\frac{1}{5}$	20. $9\frac{3}{4}+5\frac{2}{3}$	21. $7\frac{2}{5}+6\frac{2}{3}$
22. $3\frac{3}{4}+7\frac{5}{6}$	23. $2\frac{2}{5}+1\frac{1}{2}$	24. $3\frac{2}{3}+7\frac{3}{4}$	25. $8\frac{2}{5}+8\frac{3}{10}$
26. $9\frac{7}{12}+6\frac{3}{4}$	27. $2\frac{1}{2}+2\frac{2}{5}$	28. $3\frac{7}{5}+4\frac{2}{3}$	29. $2\frac{4}{9}+3\frac{2}{3}$

NOTE. Special attention should be paid to the manner of arranging work on paper, as well as to accuracy and neatness. A slovenly paper is usually indicative of a careless and inaccurate mind.

Find the sum of:

30. $2\frac{1}{2} + 2\frac{1}{3} + 3\frac{3}{4}$

31. $4\frac{1}{5} + 2\frac{1}{2} + 3\frac{3}{10}$

32. $1\frac{2}{3} + 3\frac{1}{2} + 2\frac{1}{4}$

33. $5\frac{3}{4} + 5\frac{1}{2} + 4\frac{2}{3}$

34. $4\frac{1}{3} + 2\frac{1}{6} + 1\frac{3}{4}$

35. $2\frac{1}{4} + 3\frac{1}{3} + 4\frac{5}{12}$

36. $1\frac{1}{2} + 4\frac{2}{3} + 2\frac{5}{12}$

37. $2\frac{1}{10} + 3\frac{1}{5} + 4\frac{1}{2}$

38. $4\frac{1}{6} + 2\frac{2}{3} + 1\frac{1}{2}$

39. $2\frac{3}{4} + 5\frac{1}{2} + 6\frac{7}{16}$

PROBLEMS

1. A railroad train ran the first mile in 2 minutes, the second mile in $1\frac{7}{8}$ minutes, and the third mile in $1\frac{3}{4}$ minutes. How long did it take to run the three miles?

2. Susie is $8\frac{1}{2}$ years old, Ella is $10\frac{1}{4}$ years old, and Annie is $9\frac{1}{3}$ years old. What is the sum of their ages?

3. A clerk sold $1\frac{1}{6}$ yards of cloth, $2\frac{3}{4}$ yards, and $4\frac{1}{2}$ yards. How many yards did he sell?

4. A farmer sold a calf for $\$7\frac{1}{2}$, a pig for $\$5\frac{1}{2}$, and a sheep for $\$7\frac{3}{4}$. How much money did he receive?

5. It takes $5\frac{1}{2}$ yards of braid for Mary's skirt, $3\frac{1}{3}$ yards for her waist, and $4\frac{1}{6}$ yards for her jacket. How many yards does it take for the suit?

NOTE. Care should be taken not to proceed too rapidly in the study of fractions. It takes a long time and much patient labor to lay a secure foundation. A new process should not be taken up until pupils show by their mastery of the present work that they are prepared for advanced work.

SUBTRACTION OF FRACTIONS *Oral and Written*

1. $\frac{5}{\text{apples}} - \frac{2}{\text{apples}} = ?$ **2.** $\frac{5}{\text{sevenths}} - \frac{2}{\text{sevenths}} = ?$

3. $\frac{5}{7} - \frac{2}{7} = ?$ **4.** $\frac{3}{4} - \frac{1}{4} = ?$

5. $\frac{7}{8} - \frac{3}{8} = ?$ **6.** $\frac{8}{9} - \frac{2}{9} = ?$

7. Subtract $\frac{3}{4}$ from $\frac{7}{8}$.

$$\begin{array}{r} \text{c.d.} = 8 \\ \hline \frac{7}{8} = \frac{7}{8} \\ \frac{3}{4} = \frac{6}{8} \\ \hline \frac{1}{8} \end{array}$$

Only like quantities can be subtracted. Change $\frac{3}{4}$ to 8ths. $\frac{3}{4} = \frac{6}{8}$. $\frac{7}{8} - \frac{6}{8} = \frac{1}{8}$.

To subtract one fraction from another, we express the fractions as equivalent fractions having a common denominator, and write the difference of the numerators over the common denominator.

Find the difference:

8. $\frac{1}{2} - \frac{1}{6}$	**9.** $\frac{1}{2} - \frac{1}{10}$	**10.** $\frac{1}{2} - \frac{1}{12}$	**11.** $\frac{2}{3} - \frac{1}{6}$
12. $\frac{2}{3} - \frac{4}{9}$	**13.** $\frac{2}{3} - \frac{1}{12}$	**14.** $\frac{3}{4} - \frac{5}{12}$	**15.** $\frac{5}{6} - \frac{1}{2}$
16 $\frac{5}{6} - \frac{5}{12}$	**17.** $\frac{7}{8} - \frac{7}{8}$	**18.** $\frac{8}{9} - \frac{2}{3}$	**19.** $\frac{7}{10} - \frac{1}{2}$
20. $\frac{9}{10} - \frac{2}{5}$	**21.** $\frac{2}{3} - \frac{2}{15}$	**22.** $\frac{2}{3} - \frac{2}{9}$	**23.** $\frac{1}{2} - \frac{5}{14}$
24. $\frac{7}{15} - \frac{1}{3}$	**25.** $\frac{2}{3} - \frac{5}{9}$	**26.** $\frac{11}{15} - \frac{1}{3}$	**27.** $\frac{7}{15} - \frac{1}{5}$

Find the difference:

28. $\frac{5}{6} - \frac{3}{4}$	**29.** $\frac{3}{4} - \frac{2}{3}$	**30.** $\frac{3}{4} - \frac{1}{6}$	**31.** $\frac{2}{3} - \frac{1}{2}$
32. $\frac{2}{3} - \frac{2}{5}$	**33.** $\frac{1}{2} - \frac{1}{7}$	**34.** $\frac{2}{3} - \frac{1}{4}$	**35.** $\frac{2}{3} - \frac{1}{6}$
36. $\frac{4}{5} - \frac{1}{2}$	**37.** $\frac{2}{5} - \frac{1}{3}$	**38.** $\frac{3}{4} - \frac{3}{10}$	**39.** $\frac{3}{4} - \frac{1}{6}$
40. $\frac{4}{5} - \frac{2}{3}$	**41.** $\frac{2}{3} - \frac{1}{6}$	**42.** $\frac{5}{7} - \frac{1}{2}$	**43.** $\frac{2}{3} - \frac{3}{5}$
44. $\frac{5}{6} - \frac{1}{4}$	**45.** $\frac{3}{4} - \frac{1}{3}$	**46.** $\frac{5}{6} - \frac{3}{4}$	**47.** $\frac{3}{5} - \frac{1}{3}$

48. From $\frac{3}{4}$ yr. take $\frac{1}{2}$ yr. **49.** Take \$$\frac{2}{5}$ from \$$\frac{7}{10}$.

50. From $\frac{3}{4}$ bu. take $\frac{3}{8}$ bu. **51.** Take $\frac{1}{2}$ acre from $\frac{3}{5}$ acre.

52. From $\frac{2}{3}$ hr. take $\frac{1}{2}$ hr. **53.** Take $\frac{2}{3}$ yd. from $\frac{3}{4}$ yd.

PROBLEMS

1. Mary has $\frac{7}{8}$ of a yard of ribbon. She gives $\frac{1}{4}$ of a yard to her sister. How much has she left?

2. A man buys $\frac{3}{4}$ of a ton of coal. After using $\frac{2}{3}$ of a ton, how much has he left?

3. James walks $\frac{1}{2}$ of a mile to school, and William walks $\frac{1}{6}$ of a mile. James walks how much farther than William?

4. A grocer buys eggs for $\frac{2}{6}$ of a dollar a dozen, and sells them for $\frac{1}{2}$ of a dollar. How much does he make?

5. Mr. Merrill had $\frac{5}{8}$ of a bushel of potatoes. $\frac{1}{4}$ of a bushel decayed. What part of a bushel was good?

SUBTRACTION OF MIXED NUMBERS *Written*

1. Subtract $6\frac{3}{4}$ from $9\frac{7}{12}$.

c.d. $= 12$

$9\frac{7}{12} = 9\frac{7}{12} = 8\frac{19}{12}$

$6\frac{3}{4} = 6\frac{9}{12} = 6\frac{9}{12}$

$2\frac{10}{12} = 2\frac{5}{6}$

Since $\frac{9}{12}$ cannot be taken from $\frac{7}{12}$, we take one of the 9 units in the minuend, change it to 12ths, $1 = \frac{12}{12}$, and add it to the $\frac{7}{12}$, making $\frac{19}{12}$. $9\frac{7}{12} = 8\frac{19}{12}$.

Find differences:

2. $3\frac{3}{4} - 2\frac{1}{4}$ **3.** $4\frac{1}{4} - 1\frac{3}{4}$ **4.** $3\frac{1}{6} - 1\frac{5}{6}$ **5.** $4\frac{2}{5} - 1\frac{4}{5}$

6. $2\frac{7}{8} - 1\frac{1}{2}$ **7.** $4\frac{3}{10} - 1\frac{2}{5}$ **8.** $4\frac{5}{9} - 1\frac{2}{3}$ **9.** $5\frac{1}{2} - 2\frac{3}{14}$

10. $8\frac{2}{3} - 5\frac{1}{6}$ 11. $6\frac{1}{4} - 5\frac{3}{8}$ 12. $4\frac{5}{12} - 2\frac{1}{4}$ 13. $6\frac{2}{3} - 2\frac{5}{12}$

14. $6\frac{2}{15} - 2\frac{1}{5}$ 15. $6\frac{1}{2} - 2\frac{1}{5}$ 16. $12\frac{1}{2} - 5\frac{1}{7}$ 17. $7\frac{1}{5} - 2\frac{1}{8}$

18. $10\frac{3}{4} - 2\frac{2}{3}$ 19. $8\frac{1}{3} - 4\frac{1}{2}$ 20. $7\frac{2}{3} - 2\frac{1}{2}$ 21. $9\frac{2}{5} - 5\frac{1}{2}$

PROBLEMS

1. A farmer had $10\frac{1}{6}$ dozen eggs. He sold $8\frac{1}{2}$ dozen. How many dozen had he left?

2. Mrs. Street earns $\$10\frac{1}{2}$ a week and spends $\$8\frac{2}{5}$. How much does she save?

3. A fruit dealer bought 7 boxes of oranges. After selling $2\frac{2}{5}$ boxes, how many had he left?

4. Margaret bought $12\frac{2}{3}$ yards of ribbon. After using $4\frac{1}{4}$ yards, how much has she left?

5. Mr. Price planted $8\frac{2}{3}$ acres of corn and $5\frac{1}{5}$ acres of wheat. How many more acres of corn than wheat did he plant?

REVIEW EXERCISE *Oral and Written*

1. Write two fractions that can be added or subtracted without changing their form.

2. Write two fractions that cannot be added or subtracted without changing their form.

3. Change the form of these fractions without changing their value: $\frac{2}{3}$. $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{12}$

4. Find the sum of $\frac{3}{4} + \frac{7}{12}$; $\frac{1}{2} + \frac{2}{3}$; $\frac{2}{3} + \frac{5}{6}$; $\frac{2}{3} + \frac{3}{4} + \frac{5}{6}$.

5. Add: $\frac{3}{4}$ mile + $\frac{3}{8}$ mile + $\frac{1}{2}$ mile; $\frac{2}{3}$ yd. + $\frac{5}{6}$ yd. + $\frac{1}{2}$ yd.

6. Add: $2\frac{3}{4} + 5\frac{1}{2} + 6\frac{7}{8}$; $5\frac{3}{5} + 5\frac{1}{2} + 4\frac{3}{10} + 3$.

7. What is the difference between $\frac{5}{6}$ and $\frac{5}{12}$? $\frac{1}{2}$ and $\frac{1}{7}$? $\frac{5}{12}$ and $\frac{3}{4}$? $\frac{1}{2}$ and $\frac{7}{10}$?

8. Subtract $\frac{1}{2}$ hr. from $\frac{3}{8}$ hr.; $\frac{3}{8}$ acre from $\frac{3}{4}$ acre.

9. From 4 take $\frac{3}{4}$. Express one of the units as 4ths. Then $4 = 3\frac{4}{4}$. $3\frac{4}{4} - \frac{3}{4} = 3\frac{1}{4}$.

10. From $3\frac{7}{8}$ take $1\frac{1}{2}$. Take $5\frac{3}{8}$ from $6\frac{1}{4}$.

11. Which is larger, $\frac{4}{5}$ of a dollar or $\frac{7}{10}$ of a dollar? How much larger?

12. What must be added to $\frac{3}{5}$ to make $\frac{7}{10}$?

13. What must be taken from $\frac{5}{6}$ to leave $\frac{1}{4}$?

14. From 5 take $\frac{2}{5}$. Take $\frac{7}{9}$ from 7.

15. From 6 take each of these fractions:

$\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{2}{5}$ $\frac{4}{5}$ $\frac{5}{6}$ $\frac{3}{8}$ $\frac{7}{8}$ $\frac{2}{9}$ $\frac{7}{9}$

Read:

16. $\frac{3}{4}$ $\frac{7}{7}$ $\frac{5}{3}$ $\frac{5}{8}$ $\frac{9}{9}$ $\frac{9}{8}$ $\frac{7}{10}$ $\frac{5}{5}$ $\frac{9}{5}$ $\frac{13}{12}$

17. $\frac{3}{9}$ $\frac{11}{6}$ $\frac{8}{8}$ $\frac{3}{7}$ $\frac{5}{11}$ $\frac{3}{2}$ $\frac{17}{15}$ $\frac{18}{25}$ $\frac{13}{13}$ $\frac{27}{20}$

Select from fractions in 16 and 17:

18. All the proper fractions, and name the smallest fraction that must be added to each to make it an improper fraction.

19. All the improper fractions that are equal to 1.

20. All the improper fractions that are greater than 1, and name the fraction that must be taken from each to leave 1.

NOTE. As pupils progress, all processes should be reviewed frequently.

MULTIPLICATION OF FRACTIONS *Oral and Written*

1. 2 times $\frac{3}{\text{apples}} = ?$ **2.** $2 \times \frac{3}{\text{fifths}} = ?$ **3.** $2 \times \frac{3}{5} = \frac{6}{5} = 1\frac{1}{5}$

Find:

4. $3 \times \frac{5}{7}$	**5.** $4 \times \frac{2}{3}$	**6.** $3 \times \frac{3}{8}$	**7.** $4 \times \frac{3}{5}$
8. $5 \times \frac{3}{4}$	**9.** $6 \times \frac{3}{7}$	**10.** $9 \times \frac{2}{5}$	**11.** $5 \times \frac{5}{6}$
12. $7 \times \frac{5}{8}$	**13.** $8 \times \frac{5}{9}$	**14.** $9 \times \frac{8}{10}$	**15.** $5 \times \frac{7}{9}$
16. $7 \times \frac{4}{5}$	**17.** $4 \times \frac{4}{7}$	**18.** $5 \times \frac{8}{9}$	**19.** $9 \times \frac{7}{10}$
20. $4 \times \frac{9}{11}$	**21.** $6 \times \frac{4}{13}$	**22.** $5 \times \frac{7}{12}$	**23.** $8 \times \frac{2}{15}$
24. $10 \times \frac{2}{3}$	**25.** $15 \times \frac{7}{8}$	**26.** $12 \times \frac{6}{7}$	**27.** $17 \times \frac{3}{4}$
28. $12 \times \frac{3}{5}$	**29.** $11 \times \frac{8}{15}$	**30.** $15 \times \frac{4}{7}$	**31.** $21 \times \frac{4}{5}$
32. $20 \times \frac{5}{7}$	**33.** $17 \times \frac{5}{8}$	**34.** $14 \times \frac{4}{5}$	**35.** $13 \times \frac{7}{15}$
36. $14 \times \frac{8}{9}$	**37.** $18 \times \frac{4}{7}$	**38.** $14 \times \frac{5}{9}$	**39.** $22 \times \frac{7}{9}$
40. $13 \times \frac{8}{11}$	**41.** $23 \times \frac{3}{8}$	**42.** $19 \times \frac{5}{12}$	**43.** $24 \times \frac{9}{13}$

PROBLEMS

1. A family uses $\frac{3}{5}$ of a bushel of apples a month. How many bushels will they use in 6 months?

2. If a yard of silk costs $\frac{3}{4}$ of a dollar, what will 5 yards cost?

3. If a pound of butter costs $\frac{2}{5}$ of a dollar, what will 4 pounds cost?

4. A horse eats $\frac{3}{4}$ of a peck of oats a day. How many pecks will he eat in 9 days?

5. John walks $\frac{3}{8}$ of a mile to school. How many miles does he walk in 5 mornings?

6. Prescott's hens lay $\frac{2}{3}$ of a dozen eggs every day. How many dozen do they lay in a week?

7. Mabel bought 3 hair ribbons, each $\frac{5}{8}$ of a yard long. How many yards of ribbon did she buy?

8. At $\frac{4}{5}$ of a dollar a yard, what will 9 yards of poplin cost?

9. Henry paid $\frac{4}{5}$ of a dime for marbles. What would he pay for 5 times as many?

10. If 2 handkerchiefs cost $\frac{1}{3}$ of a dollar, what will a dozen cost?

MULTIPLYING A WHOLE NUMBER BY A FRACTION

Oral and Written

1. Find $\frac{1}{3}$ of 12; 24; 36; 48; 60.

2. Find $\frac{2}{3}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{6}$; $\frac{5}{6}$, of the above numbers.

3. What is $\frac{3}{8}$ of 32?

4. What is $\frac{5}{12}$ of 72?

5. What is $\frac{5}{9}$ of 63?

6. What is $\frac{7}{8}$ of 56?

7. What is $\frac{3}{5}$ of 72?

8. What is $\frac{5}{6}$ of 54?

Finding a fractional part of a number is called multiplying by a fraction.

9. Find $\frac{2}{5}$ of 9.

$\frac{1}{5}$ of 9 is $\frac{9}{5}$; $\frac{2}{5}$ of 9 is 2 times $\frac{9}{5}$, or $\frac{18}{5}$, or $3\frac{3}{5}$.

10. $\frac{2}{3}$ of 4 **11.** $\frac{2}{5}$ of 8 **12.** $\frac{5}{6}$ of 5 **13.** $\frac{3}{4}$ of 5

14. $\frac{3}{7}$ of 8 15. $\frac{8}{9}$ of 4 16. $\frac{4}{5}$ of 9 17. $\frac{6}{7}$ of 6

18. $\frac{5}{9}$ of 8 19. $\frac{3}{8}$ of 7 20. $\frac{5}{8}$ of 3 21. $\frac{3}{8}$ of 9

The sign $\times$ in the expression $\frac{2}{3} \times 7$ is equivalent to the word "of."

Find products:

22. $\frac{5}{7} \times 8$ 23. $\frac{4}{5} \times 8$ 24. $\frac{3}{5} \times 6$ 25. $\frac{5}{6} \times 7$

26. $\frac{2}{9} \times 10$ 27. $\frac{2}{3} \times 10$ 28. $\frac{3}{4} \times 11$ 29. $\frac{2}{5} \times 4$

30. $\frac{2}{5} \times 14$ 31. $\frac{5}{6} \times 11$ 32. $\frac{3}{7} \times 9$ 33. $\frac{3}{10} \times 7$

34. $\frac{3}{4} \times 17$ 35. $\frac{3}{8} \times 16$ 36. $\frac{5}{6} \times 22$ 37. $\frac{2}{5} \times 18$

38. $\frac{3}{5} \times 24$ 39. $\frac{3}{7} \times 25$ 40. $\frac{4}{5} \times 16$ 41. $\frac{5}{7} \times 12$

42. $\frac{3}{8} \times 27$ 43. $\frac{6}{7} \times 18$ 44. $\frac{5}{8} \times 15$ 45. $\frac{4}{7} \times 10$

46. $\frac{7}{8} \times 15$ 47. $\frac{2}{9} \times 40$ 48. $\frac{4}{9} \times 32$ 49. $\frac{5}{9} \times 28$

PROBLEMS

1. At 20 cents a dozen, what will $\frac{3}{4}$ of a dozen of bananas cost?

2. George picked 15 quarts of berries and sold $\frac{3}{5}$ of them. How many quarts did he sell?

3 Olive has an allowance of 10 cents a week. She saves $\frac{1}{10}$ of it. How much does she spend?

4. A butcher bought 3 pairs of chickens for 6 dollars. How much did he pay for each chicken?

5. Ethel is 18 years old and her sister is $\frac{5}{6}$ as old. How old is her sister?

6. There are 36 shade trees on a street. Seven ninths of them are maples. How many of other kinds?

7. When oranges are 60 cents a dozen, what will one cost? 4? $\frac{1}{4}$ of a dozen? $\frac{2}{3}$ of a dozen? $\frac{3}{4}$ of a dozen? $\frac{1}{2}$ of a dozen? $\frac{5}{6}$ of a dozen?

8. I have 32 raspberry bushes and $\frac{3}{8}$ as many currant bushes. How many currant bushes?

9. If $\frac{3}{7}$ of the days are stormy, how many pleasant days in 2 weeks? In 5 weeks?

10. George has 4 dollars. Henry has $\frac{4}{5}$ as much. How much has Henry?

MULTIPLYING A MIXED NUMBER BY A WHOLE NUMBER

Written

1. Multiply $2\frac{3}{5}$ by 8.

$2\frac{3}{5}$
8
$4\frac{4}{5} = 8 \times \frac{3}{5}$
$16 = 8 \times 2$
$20\frac{4}{5} = 8 \times 2\frac{3}{5}$

This means 8 times 2 + 8 times $\frac{3}{5}$.
$8 \times \frac{3}{5} = \frac{24}{5} = 4\frac{4}{5}$
$8 \times 2 = 16$
$16 + 4\frac{4}{5} = 20\frac{4}{5}$

Multiply:

2. $2\frac{3}{8}$ by 4	3. $12\frac{1}{2}$ by 7	4. $16\frac{5}{8}$ by 7
5. $37\frac{1}{2}$ by 27	6. $3\frac{3}{4}$ by 5	7. $10\frac{1}{4}$ by 9
8. $17\frac{2}{9}$ by 9	9. $16\frac{3}{8}$ by 25	10. $4\frac{5}{6}$ by 7
11. $15\frac{5}{8}$ by 5	12. $20\frac{4}{9}$ by 8	13. $87\frac{1}{2}$ by 35
14. $5\frac{3}{8}$ by 8	15. $16\frac{5}{6}$ by 7	16. $18\frac{3}{10}$ by 9
17. $66\frac{2}{3}$ by 28	18. $3\frac{7}{8}$ by 7	19. $15\frac{5}{9}$ by 5
20. $20\frac{7}{12}$ by 5	21. $80\frac{3}{8}$ by 24	22. $83\frac{1}{3}$ by 15

PROBLEMS

1. What will 6 yards of muslin cost at $8\frac{1}{3}$ cents a yard?

2. Sugar is $5\frac{3}{4}$ cents a pound. How much must be paid for 4 pounds?

3. At $6\frac{1}{4}$ cents a pound, what will 9 pounds of meat cost?

4. Mary earns $8\frac{2}{5}$ dollars a week. How much will she earn in 4 weeks?

5. At $12\frac{1}{2}$ cents apiece, what will half a dozen collars cost?

MULTIPLYING A WHOLE NUMBER BY A MIXED NUMBER

Written

1. Multiply 12 by $7\frac{2}{5}$.

$$\begin{array}{l} 12 \\ \times\ 7\frac{2}{5} \\ \hline 4\frac{4}{5} = \frac{2}{5} \text{ of } 12 \\ 84 = 7 \times 12 \\ \hline 88\frac{4}{5} = 7\frac{2}{5} \times 12 \end{array}$$

This means 7 times 12 + $\frac{2}{5}$ of 12.
$\frac{2}{5}$ of $12 = \frac{24}{5} = 4\frac{4}{5}$
$7 \times 12 = 84$
$84 + 4\frac{4}{5} = 88\frac{4}{5}$

Multiply:

2. 8 by $2\frac{2}{3}$	3. 7 by $3\frac{1}{6}$	4. 9 by $5\frac{3}{4}$
5. 12 by $2\frac{4}{5}$	6. 5 by $3\frac{3}{8}$	7. 7 by $2\frac{5}{8}$
8. 11 by $4\frac{5}{6}$	9. 9 by $4\frac{7}{8}$	10. 8 by $4\frac{2}{9}$
11. 9 by $4\frac{3}{5}$	12. 10 by $3\frac{5}{7}$	13. 7 by $5\frac{3}{8}$
14. 12 by $8\frac{3}{4}$	15. 20 by $4\frac{3}{5}$	16. 18 by $3\frac{5}{6}$
17. 24 by $5\frac{5}{8}$	18. 17 by $2\frac{4}{5}$	19. 14 by $4\frac{3}{8}$
20. 16 by $4\frac{2}{3}$	21. 25 by $6\frac{3}{10}$	22. 36 by $5\frac{3}{5}$

MULTIPLYING A FRACTION BY A FRACTION

Oral and Written

$\frac{2}{3} \times \frac{3}{5}$ means $\frac{2}{3}$ of $\frac{3}{5}$.

1. $\frac{2}{3}$ of $\frac{3}{\text{apples}} = ?$
2. $\frac{1}{3}$ of $\frac{3}{\text{apples}} = \frac{1}{\text{apple}}$
3. $\frac{2}{3}$ of $\frac{3}{\text{apples}} = \frac{2}{\text{apples}}$
4. $\frac{2}{3}$ of $\frac{3}{5} = ?$
5. $\frac{1}{3}$ of $\frac{3}{5} = \frac{1}{5}$
6. $\frac{2}{3}$ of $\frac{3}{5} = \frac{2}{5}$

Notice that $\frac{2}{3}$ of $\frac{3}{5} = \frac{2 \times 3}{3 \times 5} = \frac{6}{15} = \frac{2}{5}$

and $\frac{2}{\cancel{3}} \times \frac{\cancel{3}}{5} = \frac{2}{5}$

To multiply a fraction by a fraction, we write the product of the numerators over the product of the denominators, canceling when possible.

This rule applies to all cases of multiplication of fractions, for every whole number may be written as a fraction with 1 for its denominator. Thus, $8 = \frac{8}{1}$; $\frac{2}{3}$ of 8 may be written $\frac{2}{3} \times \frac{8}{1}$; 8 times $\frac{2}{3}$ may be written $\frac{8}{1} \times \frac{2}{3}$.

Find:

7. $\frac{1}{4}$ of $\frac{4}{5}$	8. $\frac{1}{5}$ of $\frac{5}{6}$	9. $\frac{1}{3}$ of $\frac{6}{7}$	10. $\frac{2}{3}$ of $\frac{6}{7}$
11. $\frac{2}{3} \times \frac{3}{4}$	12. $\frac{3}{5} \times \frac{5}{6}$	13. $\frac{3}{7} \times \frac{7}{8}$	14. $\frac{3}{4} \times \frac{4}{5}$
15. $\frac{1}{2} \times \frac{1}{2}$	16. $\frac{2}{3} \times \frac{1}{4}$	17. $\frac{3}{5} \times \frac{7}{9}$	18. $\frac{9}{10} \times \frac{5}{8}$
19. $\frac{2}{3} \times \frac{7}{8}$	20. $\frac{3}{4} \times \frac{5}{6}$	21. $\frac{5}{7} \times \frac{1}{15}$	22. $\frac{8}{9} \times \frac{1}{2}$
23. $\frac{3}{4} \times \frac{1}{8}$	24. $\frac{2}{3} \times \frac{1}{2}$	25. $\frac{4}{5} \times \frac{1}{4}$	26. $\frac{7}{8} \times \frac{1}{2}$

Find the product of:

27. $\frac{1}{4} \times \frac{8}{15}$ 28. $\frac{1}{4} \times \frac{12}{17}$ 29. $\frac{2}{7} \times \frac{2}{7}$ 30. $\frac{5}{6} \times \frac{24}{25}$

31. $\frac{3}{10} \times \frac{5}{6}$ 32. $\frac{8}{15} \times \frac{3}{8}$ 33. $\frac{5}{8} \times \frac{3}{4}$ 34. $\frac{5}{21} \times \frac{14}{25}$

35. $\frac{5}{8} \times \frac{8}{9}$ 36. $\frac{3}{4} \times \frac{5}{7}$ 37. $\frac{6}{7} \times \frac{4}{5}$ 38. $\frac{11}{16} \times \frac{8}{33}$

39. $\frac{14}{15} \times \frac{5}{7}$ 40. $\frac{4}{9} \times \frac{7}{12}$ 41. $\frac{2}{3} \times \frac{4}{9}$ 42. $\frac{12}{25} \times \frac{15}{16}$

43. $\frac{5}{9} \times \frac{9}{20}$ 44. $\frac{4}{5} \times \frac{7}{8}$ 45. $\frac{2}{3} \times \frac{5}{6}$ 46. $\frac{5}{14} \times \frac{14}{35}$

PROBLEMS

1. What will $\frac{1}{2}$ of a yard of silk cost at $\frac{3}{5}$ of a dollar a yard?

2. If Alfred picks $\frac{1}{2}$ of a peck of cherries and sells $\frac{3}{4}$ of them, what part of a peck does he sell?

3. Blanche had $\frac{3}{4}$ of a pound of candy. She gave $\frac{1}{3}$ of it to Susie. How much did she give to Susie?

4. Mrs. Whiting bought $\frac{7}{8}$ of a yard of ruching and used $\frac{2}{3}$ of it. What part of a yard did she use?

5. What will $\frac{2}{3}$ of a yard of lace cost at $\frac{3}{5}$ of a dollar a yard?

6. It takes Frank $\frac{3}{4}$ of an hour to mow his lawn. Herbert can mow it in $\frac{2}{3}$ the time. How long does it take Herbert?

7. Mr. Kimball bought $\frac{4}{5}$ of a ton of oats. He had to throw away $\frac{1}{6}$ of the lot. What part of a ton did he lose?

8. Two thirds of $\frac{3}{7}$ of an acre of corn is sweet corn. How much sweet corn is there?

9. I have $\frac{4}{5}$ of a dollar. If I spend $\frac{3}{4}$ of it for a book, what part of a dollar does the book cost?

10. The schoolhouse is $\frac{5}{8}$ of a mile from my home. The church is $\frac{2}{3}$ as far. What part of a mile do I walk in going to church?

MULTIPLYING A MIXED NUMBER BY A MIXED NUMBER

Written

1. Find $1\frac{2}{3} \times 1\frac{2}{5}$. Change to improper fractions.

$$1\frac{2}{3} \times 1\frac{2}{5} = \frac{\cancel{5}}{\cancel{3}} \times \frac{7}{\cancel{5}} = \frac{7}{3} = 2\frac{1}{3}$$

Find products:

2. $2\frac{1}{3} \times 1\frac{1}{5}$	**3.** $3\frac{3}{4} \times 2\frac{2}{3}$	**4.** $2\frac{2}{5} \times 1\frac{3}{4}$	**5.** $1\frac{4}{5} \times 3\frac{1}{3}$	
6. $\frac{3}{4} \times 3\frac{2}{5}$	**7.** $\frac{5}{6} \times 4\frac{1}{2}$	**8.** $\frac{7}{8} \times 2\frac{1}{3}$	**9.** $\frac{3}{5} \times 2\frac{5}{7}$	
10. $1\frac{3}{5} \times 2\frac{1}{2}$	**11.** $3\frac{3}{4} \times 1\frac{5}{7}$	**12.** $5\frac{1}{3} \times 2\frac{1}{4}$	**13.** $2\frac{1}{7} \times 2\frac{1}{10}$	
14. $2\frac{5}{6} \times \frac{3}{5}$	**15.** $4\frac{3}{5} \times \frac{3}{9}$	**16.** $3\frac{5}{6} \times \frac{7}{5}$	**17.** $5\frac{2}{3} \times \frac{5}{7}$	
18. $1\frac{5}{6} \times 3\frac{3}{5}$	**19.** $2\frac{1}{12} \times 4\frac{4}{5}$	**20.** $4\frac{2}{7} \times 1\frac{5}{9}$	**21.** $2\frac{7}{8} \times 3\frac{3}{5}$	

FINDING WHAT PART ONE NUMBER IS OF ANOTHER

Oral

1. What part of 4 dollars is 1 dollar? 2 dollars? 3 dollars?

2. Express as parts of a gallon: 2 quarts; 1 quart.

3. Express as parts of a dollar: 50 cents; 25 cents; 75 cents; 20 cents; 40 cents; 60 cents; 80 cents.

4. What part of a bushel is 1 peck? 3 pecks?

5. What part of 12 inches is 1 inch? 5 inches? 7 inches? 11 inches?

6. Express as parts of a foot: 6 inches; 3 inches; 9 inches; 4 inches; 8 inches; 2 inches; 10 inches.

Express as parts of an hour:

7. 30 minutes **8.** 15 minutes; 45 minutes

9. 20 minutes; 40 minutes **10.** 10 minutes; 50 minutes

11. 6 minutes; 18 minutes **12.** 12 minutes; 48 minutes

13. 5 minutes; 35 minutes **14.** 3 minutes; 9 minutes

15. 4 minutes; 16 minutes

What part of:

16. 24 is 8 **17.** 32 is 4 **18.** 40 is 8 **19.** 20 is 10

20. 12 is 9 **21.** 20 is 12 **22.** 25 is 10 **23.** 40 is 30

24. 84 is 7 **25.** 63 is 9 **26.** 54 is 6 **27.** 56 is 8

28. 84 is 21 **29.** 63 is 35 **30.** 54 is 36 **31.** 56 is 32

32. Elsie solves 8 of her 10 problems. What part does she solve?

33. Out of 20 words Jack misspelled 2. What part did he misspell?

34. James bought 24 newspapers. He sold 20. What part had he left?

FINDING THE WHOLE WHEN A PART IS GIVEN *Oral*

1. Four dollars is $\frac{1}{2}$ of my money. What is the whole of it?

2. Ned sold his rabbit for 30 cents. This was $\frac{3}{5}$ of what he paid. What did he pay for the rabbit?

SOLUTION. Since 30 cents is 3 fifths, 1 fifth is $\frac{1}{3}$ of 30 cents, or 10 cents; 5 fifths is 5 × 10 cents, or 50 cents.

3. 12 is $\frac{2}{3}$ of what number? 4. 9 is $\frac{3}{4}$ of what number?

5. 24 is $\frac{6}{7}$ of what number? 6. 15 is $\frac{5}{6}$ of what number?

7. 20 is $\frac{4}{5}$ of what number? 8. 28 is $\frac{4}{7}$ of what number?

9. Maggie paid 40 cents for a veil. This was $\frac{4}{5}$ of what she paid for a pin. How much did she pay for her pin?

10. In one pasture there are 10 cows. This is $\frac{2}{3}$ of the number in another pasture. How many in the second pasture?

11. A baseball team won 12 games. This was $\frac{3}{4}$ of the number played. How many games did it play?

12. 60 miles is $\frac{5}{6}$ of the distance between two cities. How far apart are the cities?

13. I have read 48 pages of a book. This is $\frac{4}{7}$ of the book. How many pages in the book?

Find the number of which:

14. 27 is $\frac{3}{5}$ 15. 60 is $\frac{5}{8}$ 16. 84 is $\frac{7}{12}$ 17. 35 is $\frac{7}{8}$

18. 96 is $\frac{2}{3}$ 19. 96 is $\frac{3}{4}$ 20. 96 is $\frac{8}{9}$ 21. 72 is $\frac{3}{4}$

22. A man spends $\frac{7}{9}$ of his yearly wages. He spends $630. How much does he earn?

DICTATION EXERCISES

1. $9 \times 8, \div 6, +4, \div 8, \times 5, \times 7, -4, \div 11, +9 = ?$
2. $8+4, \times 5, -6, \div 9, \times 7, +3, \div 9, +7, +8 = ?$
3. $63 \div 7, \times 3, +8, \div 7, +4, \times 6 -5, \div 7, +5 = ?$
4. $84 \div 12, \times 8, -2, \div 9, \times 7, -6, \div 4, \times 7, +9 = ?$
5. $48 \div 4, +3, -7, \times 4, +8, \div 8, +3, \times 7, +4 = ?$

DRILL EXERCISE

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*
1.	6	12	2	16	4	24	18	14	10	22	8	20
2.	12	24	4	32	8	48	36	28	20	44	16	40
3.	24	48	8	64	16	96	72	56	40	88	32	80
4.	36	72	12	96	24	144	108	84	60	132	48	120
5.	9	18	3	24	6	36	27	21	15	33	12	30
6.	18	36	6	48	12	72	54	42	30	66	24	60
7.	27	54	9	72	18	108	81	63	45	99	36	90
8.	15	30	5	40	10	60	45	35	25	55	20	50
9.	30	60	10	80	20	120	90	70	50	110	40	100
10.	21	42	7	56	14	84	63	49	35	77	28	70
11.	33	66	11	88	22	132	99	77	55	121	44	110

Each number in

Row 1, or column *E*, is $\frac{2}{3}$ of what number?

Row 2, or column *K*, is $\frac{4}{5}$ of what number?

Row 3, or column *D*, is $\frac{8}{9}$ of what number?

Row 4, or column *F*, is $\frac{1}{2}$ of what number?

Row 5, or column *A*, is $\frac{3}{4}$ of what number?

Row 6, or column *B*, is $\frac{6}{7}$ of what number?

Row 7, or column *G*, is $\frac{9}{10}$ of what number?

Row 8, or column *I*, is $\frac{5}{6}$ of what number?

Row 9, or column *L*, is $\frac{10}{11}$ of what number?

Row 10, or column *H*, is $\frac{7}{8}$ of what number?

Row 11, or column *J*, is $\frac{11}{12}$ of what number?

Each number in column *C* is $\frac{1}{2}$ of what number? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{10}$? $\frac{1}{11}$? $\frac{1}{12}$?

REVIEW EXERCISE *Written*

1. Using the numbers 7 and 8, write a proper fraction; an improper fraction.

2. Write an improper fraction that can be changed to a whole number. Change it.

3. Write a proper fraction that can be changed to lower terms. Change it.

4. Write an improper fraction that can be expressed as a mixed number. Write the mixed number.

5. Write five fractions that are each equal to $\frac{2}{3}$.

6. A man said he owned $\frac{20}{32}$ of a mill. What simpler fraction might he have used?

7. Write a fraction that will show what part of the days of a week you attend school.

8. Using any of the numbers from 1 to 10, write the largest proper fraction you can; the smallest proper fraction.

9. Using the numbers 3, 4, 4, 5, write two fractions that can be added without changing their form. Add them.

10. Using the same numbers, write two fractions that cannot be added without changing their form. Change their form and add them.

11. Write as fractions: 1, 5, 8.

12. From 8 take $\frac{1}{8}$, $1\frac{1}{8}$; $2\frac{7}{8}$; $2\frac{3}{4}$, $3\frac{1}{2}$.

13. Find 4 times $\frac{5}{7}$; $\frac{7}{8}$, $2\frac{1}{4}$; $2\frac{3}{5}$; $3\frac{2}{9}$.

14. Find $2\frac{1}{2}$ times 2, $\frac{3}{5}$; $1\frac{2}{5}$; $1\frac{1}{2}$; $2\frac{5}{8}$.

15. Find $\frac{1}{2}$ of $\frac{3}{4}$; $\frac{6}{7}$; $\frac{1}{2}$; $1\frac{1}{2}$; $2\frac{3}{5}$.

16. Find $\frac{3}{8}$ of $\frac{3}{4}$; $\frac{7}{9}$; $1\frac{1}{2}$; 2; $2\frac{2}{3}$.

17. What is $\frac{3}{4}$ of 12?

18. 12 is $\frac{3}{4}$ of what number?

19. John sold 18 doves. This was $\frac{3}{5}$ of the number he had at first. How many had he at first?

DIVISION *Oral and Written*

1. $\frac{4}{\text{apples}} \div \frac{2}{\text{apples}} = ?$

2. $\frac{4}{\text{fifths}} \div \frac{2}{\text{fifths}} = ?$

3. $\frac{4}{5} \div \frac{2}{5} = ?$

4. $\frac{5}{8} \div \frac{1}{8} = ?$

5. $\frac{5}{8} \div \frac{2}{8} = ?$

6. $\frac{5}{8} \div \frac{3}{8} = ?$

7. $\frac{4}{5} \div \frac{2}{3} = ?$ Change to like fractions.

$1\frac{2}{15} \div 1\frac{0}{15}$

$\frac{12}{\text{fifteenths}} \div \frac{10}{\text{fifteenths}} = 12 \div 10 = 1\frac{2}{10} = 1\frac{1}{5}$

8. Divide 4 by $\frac{2}{3}$. $4 = \frac{4}{1}$, for any whole number may be expressed as a fraction with 1 for its denominator.

$\frac{4}{1} \div \frac{2}{3}$ Change to like fractions.

$1\frac{2}{3} \div \frac{2}{3} = 12 \div 2 = 6$

9. Divide $\frac{5}{6}$ by 2.

$\frac{5}{6} \div \frac{2}{1}$ Change to like fractions.

$\frac{5}{6} \div 1\frac{2}{6} = 5 \div 12 = \frac{5}{12}$

Any number may be divided by a fraction by changing both numbers to fractions having a common denominator,

and then dividing the numerator of the dividend by the numerator of the divisor.

By multiplying the dividend by the divisor inverted, we obtain the same results as in the process just described.

Thus,
$$\frac{4}{5} \div \frac{2}{3} = \frac{\overset{2}{\cancel{4}}}{5} \times \frac{3}{\underset{1}{\cancel{2}}} = \frac{6}{5} = 1\tfrac{1}{5}$$

$$4 \div \frac{2}{3} = \frac{\overset{2}{\cancel{4}}}{1} \times \frac{3}{\underset{1}{\cancel{2}}} = 6$$

$$\frac{5}{6} \div 2 = \frac{5}{6} \times \frac{1}{2} = \frac{5}{12}$$

To divide fractions, we change to like fractions and divide the numerator of the dividend by the numerator of the divisor; or, for convenience, we invert the divisor and multiply, canceling when possible.

Divide by changing to like fractions, or by inverting the divisor:

	A	*B*	*C*	*D*	*E*
10.	$4 \div \frac{1}{2}$	$\frac{1}{6} \div 2$	$9 \div \frac{3}{7}$	$\frac{2}{5} \div \frac{1}{2}$	$\frac{2}{3} \div \frac{1}{5}$
11.	$6 \div \frac{1}{3}$	$\frac{3}{4} \div 3$	$\frac{3}{4} \div 5$	$\frac{3}{4} \div \frac{3}{8}$	$\frac{4}{5} \div \frac{3}{4}$
12.	$3 \div \frac{1}{4}$	$\frac{7}{8} \div 7$	$12 \div \frac{4}{5}$	$\frac{3}{5} \div \frac{5}{8}$	$\frac{2}{3} \div \frac{3}{4}$
13.	$5 \div \frac{1}{6}$	$\frac{4}{5} \div 6$	$\frac{5}{6} \div \frac{1}{4}$	$\frac{5}{8} \div \frac{5}{7}$	$\frac{6}{7} \div \frac{2}{3}$
14.	$2 \div \frac{1}{8}$	$\frac{5}{7} \div 5$	$\frac{3}{8} \div \frac{3}{4}$	$\frac{4}{9} \div \frac{5}{6}$	$\frac{3}{5} \div \frac{3}{4}$
15.	$8 \div \frac{3}{4}$	$\frac{8}{9} \div 3$	$\frac{3}{4} \div \frac{7}{8}$	$\frac{3}{5} \div \frac{6}{7}$	$\frac{3}{8} \div \frac{7}{8}$
16.	$5 \div \frac{2}{5}$	$\frac{4}{5} \div 2$	$\frac{2}{3} \div \frac{8}{9}$	$\frac{8}{9} \div \frac{2}{3}$	$\frac{4}{9} \div \frac{2}{3}$
17.	$10 \div \frac{5}{6}$	$\frac{3}{8} \div 5$	$\frac{4}{5} \div \frac{2}{3}$	$\frac{2}{3} \div \frac{5}{6}$	$\frac{5}{6} \div \frac{3}{4}$

DIVIDING MIXED NUMBERS *Written*

1. Divide $9\frac{2}{7}$ by 4.

(1) $9\frac{2}{7} \div 4 = \frac{65}{7} \div 4 = \frac{65}{7} \times \frac{1}{4} = \frac{65}{28} = 2\frac{9}{28}$

(2) $4)9\frac{2}{7}$ $\frac{1}{4}$ of 9 is 2, and 1 over

$2\frac{9}{28}$ $\frac{1}{4}$ of $1\frac{2}{7} = \frac{1}{4}$ of $\frac{9}{7} = \frac{9}{28}$

Give quotients:

2.	$3\frac{1}{5} \div 2$	**3.**	$16\frac{2}{3} \div 7$	**4.**	$16\frac{2}{3} \div 4$	**5.**	$14\frac{4}{5} \div 6$		
6.	$4\frac{7}{8} \div 3$	**7.**	$8\frac{1}{3} \div 5$	**8.**	$14\frac{2}{3} \div 5$	**9.**	$12\frac{2}{5} \div 7$		
10.	$8\frac{2}{3} \div 4$	**11.**	$8\frac{1}{2} \div 6$	**12.**	$15\frac{5}{8} \div 4$	**13.**	$24\frac{3}{4} \div 3$		
14.	$12\frac{1}{2} \div 3$	**15.**	$12\frac{2}{5} \div 7$	**16.**	$33\frac{1}{3} \div 8$	**17.**	$14\frac{4}{5} \div 6$		
18.	$9\frac{5}{7} \div 5$	**19.**	$7\frac{2}{3} \div 5$	**20.**	$18\frac{3}{4} \div 2$	**21.**	$33\frac{1}{3} \div 8$		
22.	$20\frac{2}{3} \div 6$	**23.**	$15\frac{7}{9} \div 4$	**24.**	$17\frac{7}{8} \div 7$	**25.**	$25\frac{1}{2} \div 9$		

26. Divide $3\frac{1}{2}$ by $1\frac{3}{4}$.

$$3\frac{1}{2} \div 1\frac{3}{4} = \frac{7}{2} \div \frac{7}{4} = \frac{7}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{7} = 2$$

Give quotients:

27.	$2\frac{1}{4} \div 1\frac{1}{8}$	**28.**	$\frac{7}{8} \div 1\frac{5}{8}$	**29.**	$\frac{3}{4} \div 1\frac{1}{2}$	**30.**	$1\frac{2}{3} \div \frac{5}{9}$		
31.	$1\frac{1}{2} \div 2\frac{1}{4}$	**32.**	$1\frac{1}{4} \div \frac{2}{5}$	**33.**	$3\frac{1}{8} \div \frac{5}{8}$	**34.**	$\frac{7}{8} \div 2\frac{1}{3}$		
35.	$3\frac{1}{3} \div \frac{2}{5}$	**36.**	$\frac{5}{6} \div 2\frac{1}{4}$	**37.**	$\frac{2}{3} \div 3\frac{1}{3}$	**38.**	$3\frac{3}{4} \div 1\frac{7}{8}$		
39.	$2\frac{2}{5} \div 5\frac{2}{5}$	**40.**	$3\frac{3}{5} \div 2\frac{1}{4}$	**41.**	$1\frac{2}{3} \div 3\frac{2}{3}$	**42.**	$5\frac{2}{5} \div 2\frac{1}{4}$		
43.	$4\frac{1}{8} \div 1\frac{1}{2}$	**44.**	$2\frac{3}{4} \div 1\frac{5}{6}$	**45.**	$6\frac{1}{4} \div 1\frac{7}{8}$	**46.**	$2\frac{2}{7} \div 1\frac{1}{8}$		

PROBLEMS *Written*

1. If 1 pound of coffee costs $\frac{1}{4}$ of a dollar, how many pounds can be bought for $6\frac{1}{2}$ dollars?

2. Mrs. Martin paid $4\frac{3}{5}$ dollars for $5\frac{1}{2}$ yards of cloth. What was the price a yard?

3. Two boys walked 4 miles in $2\frac{3}{4}$ hours. How far did they walk in 1 hour?

4. Alice paid the photographer $1\frac{2}{5}$ dollars for finishing 20 pictures. What was the cost of each picture?

5. At $5\frac{1}{2}$ cents a pound, how many pounds of sugar can be bought for 55 cents?

6. If 9 yards of carpeting cost $12\frac{3}{5}$ dollars, what will 1 yard cost?

7. A boy picked 4 boxes of strawberries and sold them for 50 cents. How much did he receive a box?

8. A farmer's coal cost 33 dollars. He paid for it in apples worth $2\frac{3}{4}$ dollars a barrel. How many barrels did it take?

9. If a man earns $1\frac{4}{5}$ dollars a day, how long will it take him to earn 9 dollars?

10. Three barrels of flour cost $19\frac{1}{2}$ dollars. What was the price of a barrel?

11. A field containing $2\frac{5}{8}$ acres is cut up into 7 equal lots. What part of an acre is each lot?

12. Mrs. Jones sold some eggs for $\frac{2}{5}$ of a dollar a dozen. She received $1\frac{3}{5}$ dollars for them. How many dozen did she sell?

13. If $4\frac{1}{5}$ dollars will pay for 7 pounds of tea, what is the cost of a pound?

14. In 6 minutes a railroad train ran $4\frac{2}{7}$ miles. What was the rate per minute?

15. A woman received $11\frac{1}{4}$ dollars for 5 days' work. What did she receive a day?

16. At $\frac{2}{5}$ of a dollar a pound, how many pounds of chocolate can be bought for $2\frac{2}{5}$ dollars?

17. How many half-gallon bottles will be required to bottle $3\frac{1}{2}$ gallons of vinegar?

18. How many strips of paper $\frac{5}{8}$ of a yard wide will be needed to cover the side of a room 5 yards long?

19. Julia uses $\frac{4}{9}$ of a yard of cretonne to make a work-bag. How many bags can she make from 4 yards?

20. Mrs. Danforth divided $3\frac{1}{2}$ pounds of candy among 4 children. What part of a pound did she give to each child?

REVIEW EXERCISE *Oral and Written*

1. Name the largest of these quantities: $\frac{1}{8}$ of a dollar, $\frac{1}{3}$ of a dollar, $\frac{1}{2}$ of a dollar.

2. What is the product of $10 \times \frac{3}{5}$? $12 \times \frac{6}{7}$? $20 \times \frac{5}{12}$?

3. What does the expression $\frac{4}{5} \times 9$ mean?

4. Find the product of $\frac{4}{5} \times 8$; $\frac{3}{7} \times 9$; $\frac{3}{8} \times 2$.

5. Multiply $3\frac{3}{4}$ by 5; $5\frac{3}{8}$ by 8; $12\frac{1}{2}$ by 7.

6. Multiply 8 by $2\frac{2}{3}$; 12 by $4\frac{2}{9}$; 20 by $3\frac{3}{5}$.

7. What is $\frac{1}{4}$ of $\frac{8}{15}$? $\frac{5}{9}$ of $\frac{9}{20}$? $\frac{2}{3}$ of $\frac{4}{5}$?

8. Find $2\frac{3}{5} \times 2\frac{1}{2}$; $\frac{5}{6} \times 4\frac{1}{2}$; $2\frac{2}{7} \times 2\frac{1}{10}$.

9. Divide $\frac{2}{3}$ by $\frac{5}{6}$; $\frac{3}{9}$ by $\frac{4}{5}$; $\frac{9}{10}$ by $\frac{5}{6}$.

10. Divide 9 by $\frac{3}{7}$; 12 by $\frac{4}{5}$; $\frac{5}{6}$ by 4; $\frac{7}{8}$ by 7.

11. Divide $3\frac{3}{5}$ by $2\frac{1}{4}$; $4\frac{1}{5}$ by $\frac{5}{7}$; $\frac{4}{7}$ by $3\frac{3}{7}$; $4\frac{3}{8}$ by 5.

12. What is the value of $\frac{5}{6} + \frac{3}{4}$? $\frac{5}{6} - \frac{3}{4}$? $\frac{5}{6}$ of $\frac{3}{4}$? $\frac{5}{6} \div \frac{3}{4}$?

MISCELLANEOUS PROBLEMS *Written*

1. What is the cost of $3\frac{3}{4}$ pounds of coffee at $\frac{2}{5}$ of a dollar a pound?

Omitting fractions, read "What will 3 pounds cost at 1 dollar a pound?" 3 times 1 dollar.

Similarly, $3\frac{3}{4}$ pounds will cost $3\frac{3}{4}$ times $\frac{2}{5}$ of a dollar.

$$3\tfrac{3}{4} \times \frac{2}{5} = \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{4}}} \times \frac{\cancel{2}}{\cancel{5}} = \frac{3}{2}, \text{ or } 1\tfrac{1}{2} \qquad \text{Answer, } \$1\tfrac{1}{2}.$$

Notice that in the mechanical work we treat the quantities as abstract numbers.

2. What must I pay for $2\frac{1}{2}$ tons of coal at $6\frac{4}{5}$ dollars a ton?

3. A bushel of oats weighs 32 pounds. What is the weight of a load of $20\frac{5}{8}$ bushels?

4. Mr. Farmer has 280 sheep. Mr. Harlow has $2\frac{1}{8}$ times as many. How many has Mr. Harlow?

5. How many quarts of pickles are there in 15 jars if each jar holds $1\frac{3}{4}$ quarts?

6. At 12 cents a pound, how much must be paid for 6 cheeses, each weighing $12\frac{5}{8}$ pounds?

7. If $15\frac{5}{8}$ yards of cloth cost $6\frac{1}{4}$ dollars, what is the cost of 1 yard?

Omitting fractions, read "If 15 yards cost 6 dollars, what will 1 yard cost?" $\$6 \div 15 =$ cost of 1 yard.

Similarly, $\$6\frac{1}{4} \div 15\frac{5}{8} =$ cost of 1 yard.

$$6\tfrac{1}{4} \div 15\tfrac{5}{8} = \frac{25}{4} \times \frac{8}{125} = \frac{2}{5}$$

Answer, $\frac{2}{5}$ of a dollar.

8. How many bushels of potatoes at $\frac{4}{5}$ of a dollar a bushel can be bought for 20 dollars?

9. In 6 days James earned $\$10\frac{1}{2}$. What were his daily wages?

10. For $5\frac{1}{2}$ days' work a gardener received $13\frac{3}{4}$ dollars. How much did he receive a day?

11. It takes $\frac{5}{8}$ of a yard of cloth to make an apron. How many aprons can be made from $7\frac{1}{2}$ yards of cloth?

12. If $\frac{7}{8}$ of a yard of cloth is used for an apron, how many yards must be bought to make 20 aprons?

13. How much cloth is used for an apron when 22 aprons are made from $8\frac{1}{4}$ yards?

14. A small park contains $5\frac{3}{5}$ acres. In the same city there is another park $8\frac{3}{4}$ times as large. What is the size of the larger park?

15. A clerk receives $\$60$ a month. He spends $\$20\frac{3}{4}$ for board, $\$7\frac{1}{2}$ for room rent, $\$5\frac{1}{4}$ for clothing, and $\$1\frac{3}{4}$ for car fares. How much does he save?

16. A carpenter agreed to do a piece of work at $\$3\frac{3}{4}$ a day. He worked $7\frac{1}{2}$ days. How much did he charge?

17. Oil is worth at the wells 37½ cents a barrel. What are 1000 barrels worth?

18. Mr. Jenkins received $108½ for his apples. He sold them at $1¾ a barrel. How many barrels did he sell?

19. From 6½ acres of land there were cut 9¾ tons of hay. What was the yield of one acre?

20. A can contains 8½ quarts of milk. How much is left after 1½ quarts are sold to one customer and twice as much to another customer?

RELATION OF ONE NUMBER TO ANOTHER *Oral*

1. What is the relation of 8 to 2?

The relation of 8 to 2 is found by dividing 8 by 2. $8 \div 2 = 4$.

The relation of one number to another is called their *ratio*.

This principle is nothing new, as every division expresses a ratio, as, also, does every fraction.

Ratio is expressed by the sign : written between the two numbers or quantities. This sign is equivalent to the sign of division, and means that the first number is to be divided by the second. $5)\,3$, $3 \div 5$, $\frac{3}{5}$, 3 : 5, and the ratio 3 to 5, all mean the same thing.

2. What is the ratio (1) of 12 to 3; (2) of 3 to 12?

(1) The ratio of 12 to $3 = \frac{12}{3} = 4$.

(2) The ratio of 3 to $12 = \frac{3}{12} = \frac{1}{4}$.

Find the ratio of:

3. 20 to 4	**4.** 27 to 9	**5.** 2 to 10	**6.** 3 to 15
7. 54 : 9	**8.** 56 : 7	**9.** 12 : 60	**10.** 5 : 40

11. 36 : 6 **12.** 35 : 5 **13.** 8 : 48 **14.** 7 : 63

15. 28 : 7 **16.** 84 : 7 **17.** 12 : 72 **18.** 8 : 32

What is the ratio of:

19. 56 days to 8 days **20.** 5 boys to 60 boys

21. 32 men to 4 men **22.** 8 barrels to 48 barrels

23. Mr. Rich is 40 years old. His son Harry is 8 years old. What is the ratio of the father's age to the son's age?

24. Harriet solved 9 out of 10 problems. What is the ratio of the number solved to the number given?

25. Jennie has 2 dolls. Maggie has 6. What is the ratio of Jennie's dolls to Maggie's dolls?

26. A shoe dealer sold 40 pairs of shoes in the afternoon and 20 pairs in the evening. What is the ratio of the afternoon sales to the evening sales?

The following table contains pairs of fractions whose sums, differences, and quotients have no denominators greater than 16. The exercises should be used frequently for a few moments at a time for quick oral work until pupils acquire accuracy and facility in the use of these simple fractions.

1. Add the fractions in each couplet.

2. Subtract the second fraction in each couplet from the first fraction.

3. Find the product of the fractions in each couplet.

4. Divide the first fraction in each couplet by the second fraction.

5. Compare the first fraction in each couplet with the second fraction.

6. Compare the second fraction in each couplet with the first fraction.

7. Make up simple problems based upon the fractions given in the table.

DRILL TABLE IN FRACTIONS

	A	*B*	*C*	*D*	*E*	*F*
1.	$\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{4}$	$\frac{3}{4}$ $\frac{1}{2}$			
2.	$\frac{1}{2}$ $\frac{1}{3}$	$\frac{2}{3}$ $\frac{1}{2}$	$\frac{1}{3}$ $\frac{1}{6}$	$\frac{2}{3}$ $\frac{1}{6}$	$\frac{5}{6}$ $\frac{1}{3}$	$\frac{5}{6}$ $\frac{2}{3}$
3.	$\frac{1}{2}$ $\frac{1}{6}$	$\frac{5}{6}$ $\frac{1}{2}$				
4.	$\frac{1}{4}$ $\frac{1}{8}$	$\frac{3}{4}$ $\frac{1}{8}$	$\frac{3}{8}$ $\frac{1}{4}$	$\frac{5}{8}$ $\frac{1}{4}$	$\frac{7}{8}$ $\frac{1}{4}$	$\frac{3}{4}$ $\frac{3}{8}$
5.	$\frac{3}{4}$ $\frac{5}{8}$	$\frac{7}{8}$ $\frac{3}{4}$	$\frac{1}{2}$ $\frac{1}{8}$	$\frac{1}{2}$ $\frac{3}{8}$	$\frac{5}{8}$ $\frac{1}{2}$	$\frac{7}{8}$ $\frac{1}{2}$
6.	$\frac{1}{3}$ $\frac{1}{9}$	$\frac{1}{3}$ $\frac{2}{9}$	$\frac{4}{9}$ $\frac{1}{3}$	$\frac{5}{9}$ $\frac{1}{3}$	$\frac{7}{9}$ $\frac{1}{3}$	$\frac{8}{9}$ $\frac{1}{3}$
7.	$\frac{2}{3}$ $\frac{1}{9}$	$\frac{2}{3}$ $\frac{2}{9}$	$\frac{2}{3}$ $\frac{4}{9}$	$\frac{2}{3}$ $\frac{5}{9}$	$\frac{7}{9}$ $\frac{2}{3}$	$\frac{8}{9}$ $\frac{2}{3}$
8.	$\frac{1}{2}$ $\frac{1}{5}$	$\frac{1}{2}$ $\frac{2}{5}$	$\frac{3}{5}$ $\frac{1}{2}$	$\frac{4}{5}$ $\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{10}$	$\frac{1}{2}$ $\frac{3}{10}$
9.	$\frac{7}{10}$ $\frac{1}{2}$	$\frac{9}{10}$ $\frac{1}{2}$	$\frac{1}{5}$ $\frac{1}{10}$	$\frac{3}{10}$ $\frac{1}{5}$	$\frac{7}{10}$ $\frac{1}{5}$	$\frac{9}{10}$ $\frac{1}{5}$
10.	$\frac{2}{5}$ $\frac{1}{10}$	$\frac{2}{5}$ $\frac{3}{10}$	$\frac{7}{10}$ $\frac{2}{5}$	$\frac{9}{10}$ $\frac{2}{5}$	$\frac{3}{5}$ $\frac{1}{10}$	$\frac{3}{5}$ $\frac{3}{10}$
11.	$\frac{7}{10}$ $\frac{3}{5}$	$\frac{9}{10}$ $\frac{3}{5}$	$\frac{4}{5}$ $\frac{1}{10}$	$\frac{4}{5}$ $\frac{3}{10}$	$\frac{4}{5}$ $\frac{7}{10}$	$\frac{9}{10}$ $\frac{4}{5}$

12.	$\frac{1}{2}$ $\frac{1}{12}$	$\frac{1}{2}$ $\frac{5}{12}$	$\frac{7}{12}$ $\frac{1}{2}$	$\frac{11}{12}$ $\frac{1}{2}$	$\frac{1}{3}$ $\frac{1}{12}$	$\frac{5}{12}$ $\frac{1}{3}$
13.	$\frac{7}{12}$ $\frac{1}{3}$	$\frac{11}{12}$ $\frac{1}{3}$	$\frac{2}{3}$ $\frac{1}{12}$	$\frac{2}{3}$ $\frac{5}{12}$	$\frac{2}{3}$ $\frac{7}{12}$	$\frac{11}{12}$ $\frac{2}{3}$
14.	$\frac{1}{4}$ $\frac{1}{12}$	$\frac{5}{12}$ $\frac{1}{4}$	$\frac{7}{12}$ $\frac{1}{4}$	$\frac{11}{12}$ $\frac{1}{4}$	$\frac{3}{4}$ $\frac{1}{12}$	$\frac{3}{4}$ $\frac{5}{12}$
15.	$\frac{3}{4}$ $\frac{7}{12}$	$\frac{11}{12}$ $\frac{3}{4}$	$\frac{1}{3}$ $\frac{1}{4}$	$\frac{3}{4}$ $\frac{1}{3}$	$\frac{2}{3}$ $\frac{1}{4}$	$\frac{3}{4}$ $\frac{2}{3}$
16.	$\frac{1}{6}$ $\frac{1}{12}$	$\frac{5}{12}$ $\frac{1}{6}$	$\frac{7}{12}$ $\frac{1}{6}$	$\frac{11}{12}$ $\frac{1}{6}$	$\frac{5}{6}$ $\frac{1}{12}$	$\frac{5}{6}$ $\frac{5}{12}$
17.	$\frac{5}{6}$ $\frac{7}{12}$	$\frac{11}{12}$ $\frac{5}{6}$				

18.	$\frac{1}{2}$ $\frac{1}{14}$	$\frac{1}{2}$ $\frac{3}{14}$	$\frac{1}{2}$ $\frac{5}{14}$	$\frac{9}{14}$ $\frac{1}{2}$	$\frac{11}{14}$ $\frac{1}{2}$	$\frac{13}{14}$ $\frac{1}{2}$
19.	$\frac{1}{2}$ $\frac{1}{7}$	$\frac{1}{2}$ $\frac{3}{7}$	$\frac{1}{2}$ $\frac{3}{7}$	$\frac{4}{7}$ $\frac{1}{2}$	$\frac{5}{7}$ $\frac{1}{2}$	$\frac{6}{7}$ $\frac{1}{2}$

20.	$\frac{1}{3}$ $\frac{1}{5}$	$\frac{2}{5}$ $\frac{1}{3}$	$\frac{3}{5}$ $\frac{1}{3}$	$\frac{4}{5}$ $\frac{1}{3}$	$\frac{2}{3}$ $\frac{1}{5}$	$\frac{2}{3}$ $\frac{2}{5}$
21.	$\frac{2}{3}$ $\frac{3}{5}$	$\frac{4}{5}$ $\frac{2}{3}$				

22.	$\frac{1}{2}$ $\frac{1}{16}$	$\frac{1}{2}$ $\frac{3}{16}$	$\frac{1}{2}$ $\frac{5}{16}$	$\frac{1}{2}$ $\frac{7}{16}$	$\frac{9}{16}$ $\frac{1}{2}$	$\frac{11}{16}$ $\frac{1}{2}$
23.	$\frac{13}{16}$ $\frac{1}{2}$	$\frac{15}{16}$ $\frac{1}{2}$	$\frac{1}{4}$ $\frac{1}{16}$	$\frac{1}{4}$ $\frac{3}{16}$	$\frac{1}{4}$ $\frac{5}{16}$	$\frac{7}{16}$ $\frac{1}{4}$
24.	$\frac{9}{16}$ $\frac{1}{4}$	$\frac{11}{16}$ $\frac{1}{4}$	$\frac{13}{16}$ $\frac{1}{4}$	$\frac{15}{16}$ $\frac{1}{4}$	$\frac{3}{4}$ $\frac{1}{16}$	$\frac{3}{4}$ $\frac{3}{16}$
25.	$\frac{3}{4}$ $\frac{5}{16}$	$\frac{3}{4}$ $\frac{7}{16}$	$\frac{3}{4}$ $\frac{9}{16}$	$\frac{3}{4}$ $\frac{11}{16}$	$\frac{13}{16}$ $\frac{3}{4}$	$\frac{15}{16}$ $\frac{3}{4}$
26.	$\frac{1}{8}$ $\frac{1}{16}$	$\frac{3}{16}$ $\frac{1}{8}$	$\frac{5}{16}$ $\frac{1}{8}$	$\frac{7}{16}$ $\frac{1}{8}$	$\frac{9}{16}$ $\frac{1}{8}$	$\frac{11}{16}$ $\frac{1}{8}$
27.	$\frac{13}{16}$ $\frac{1}{8}$	$\frac{15}{16}$ $\frac{1}{8}$	$\frac{3}{8}$ $\frac{1}{16}$	$\frac{3}{8}$ $\frac{3}{16}$	$\frac{3}{8}$ $\frac{5}{16}$	$\frac{7}{16}$ $\frac{3}{8}$
28.	$\frac{9}{16}$ $\frac{3}{8}$	$\frac{11}{16}$ $\frac{3}{8}$	$\frac{13}{16}$ $\frac{3}{8}$	$\frac{15}{16}$ $\frac{7}{8}$	$\frac{5}{8}$ $\frac{1}{16}$	$\frac{5}{8}$ $\frac{3}{16}$
29.	$\frac{5}{8}$ $\frac{5}{16}$	$\frac{5}{8}$ $\frac{7}{16}$	$\frac{5}{8}$ $\frac{9}{16}$	$\frac{11}{16}$ $\frac{5}{8}$	$\frac{13}{16}$ $\frac{5}{8}$	$\frac{15}{16}$ $\frac{5}{8}$
30.	$\frac{7}{8}$ $\frac{1}{16}$	$\frac{7}{8}$ $\frac{3}{16}$	$\frac{7}{8}$ $\frac{5}{16}$	$\frac{7}{8}$ $\frac{7}{16}$	$\frac{7}{8}$ $\frac{9}{16}$	$\frac{7}{8}$ $\frac{11}{16}$
31.	$\frac{7}{8}$ $\frac{13}{16}$	$\frac{15}{16}$ $\frac{7}{8}$				

DRILL EXERCISE IN RAPID ADDITION AND SUBTRACTION

7	4	6	1	9	5	3	8	2	6	8	5
5											9
3											7
8											2
6											6
4											3
7											8
9											6
3											7
2											5
7											8
5	3	8	9	4	7	2	6	8	7	5	9

1. Beginning with any number in the margin and going in either direction, rapidly add the numbers until 100 or any given number is reached.

2. Beginning with 100 or any given number, rapidly subtract the successive numbers in the margin.

NOTE. The above exercise is valuable only when additions and subtractions are performed rapidly.

MISCELLANEOUS PROBLEMS *Written*

1. Two men do a piece of work for 84 dollars. One does $\frac{3}{7}$ of the work. How much ought each to receive?

2. John had $\frac{15}{20}$ of a dollar. He gave $\frac{2}{5}$ of it to his sister. How much had he left?

3. If he had given $\frac{2}{5}$ of a dollar to his sister, how much would he have had left?

4. After spending $\frac{2}{3}$ of his money for a knife, Austin had 24 cents left. How much had he at first?

5. What is the cost of a yard of cloth when $\frac{1}{2}$ of a yard costs $\frac{1}{2}$ of a dollar?

6. What is the cost of $\frac{1}{2}$ of a yard of cloth at $\frac{1}{2}$ of a dollar a yard?

7. What is the cost of $1\frac{1}{2}$ yards of cloth at $1\frac{1}{2}$ dollars a yard?

8. Rope is sold for $2\frac{5}{8}$ cents a foot. How much will 176 feet cost?

9. A merchant paid $9\frac{3}{5}$ dollars for a dozen hats. He sold them at cost. How much did he receive for each hat?

10. A book which cost $\frac{9}{10}$ of a dollar was sold for $\frac{4}{5}$ of a dollar. What was the loss?

11. What are the daily wages of a man who earns $13\frac{1}{2}$ dollars in a week?

12. A telephone pole 30 feet long was set 6 feet in the ground. What part of the pole was in the ground?

13. A grocer had 72 gallons of molasses. He sold $\frac{1}{4}$ of it to one customer and $\frac{1}{3}$ of it to another. How many gallons had he left?

14. Mr. Brown has 100 dollars. If he pays the grocer $17\frac{1}{2}$ dollars, and buys 8 cords of wood at $5\frac{3}{4}$ dollars a cord, how much will he have left?

15. By the single package, raisins are 12 cents; by the dozen packages, $10\frac{3}{4}$ cents. What is saved by buying a dozen packages at a time?

16. What is the change from 2 ten-dollar bills given to pay for $2\frac{1}{2}$ tons of coal at $6\frac{2}{5}$ dollars a ton?

17. Two boys started from the same point and walked in opposite directions. One walked $3\frac{5}{8}$ miles and the other $2\frac{3}{4}$ miles. How far apart were they then?

18. What is the cost of $\frac{3}{4}$ of a pound of tea at $\frac{2}{5}$ of a dollar a pound?

19. A man earns $17\frac{1}{2}$ dollars a week and saves $\frac{3}{7}$ of it. How much can he save in 4 weeks?

20. Three fourths of a fish line is 36 feet. How long is the line?

21. Another line is $\frac{2}{3}$ of 36 feet. How long is this line?

22. What part of a dollar is 50 cents? 25 cents? 75 cents?

23. What part of a dollar is 20 cents? 40 cents? 60 cents? 80 cents?

24. What part of a dollar is 10 cents? 30 cents? 70 cents? 90 cents?

25. If land is worth 100 dollars an acre, what part of an acre can be bought with 50 dollars? 25 dollars? 75 dollars?

26. What part of a century (one hundred years) is 20 years? 40 years? 60 years? 80 years?

27. A bundle of 10 pencils is what part of 100 pencils? A bundle of 30? A bundle of 70? A bundle of 90?

28. How long will it take Joseph to save 21 dollars for a bicycle if he saves $1\frac{3}{4}$ dollars a week?

29. Every Saturday night Robert puts $\frac{3}{4}$ of a dollar in the savings bank. How much will he save in 20 weeks?

30. If you earn 2 dollars and save $\frac{3}{4}$ of it, how much do you save?

31. If you earn 2 dollars and save 1 cent out of every 10, how much do you save?

MEASURING DISTANCES *Oral and Written*

Distances have one dimension — length.

We measure short distances in inches, feet, and yards.

1. How many lines 1 inch long will make a line 1 foot long?

2. How many feet long is a yardstick?

3. How many inches make a yard?

We measure long distances in rods and miles.

4. $5\frac{1}{2}$ yards make a rod. How many feet in a rod?

5. A distance of 320 rods is a mile. How many feet in a mile?

6. Write the table of long or linear measure.

7. My desk is 54 inches long. Express its length in feet and inches. In feet.

8. One foot minus 3 inches is how many inches? What part of a foot?

9. One stick is $1\frac{1}{2}$ feet long and another 15 inches. Both together will reach how far?

10. A house lot is 4 rods long. What is its length in feet?

11. It is 32 rods around a running track in a playground. How many times will Henry and Forrest go around it in running a mile? $\frac{1}{2}$ of a mile?

12. If your steps are $1\frac{2}{3}$ feet long, how many will you take in walking a mile?

MEASURING SURFACES *Oral*

NOTE. An accurate conception of surface area is rare with young pupils. A little time spent now in developing this idea will prove of great help in subsequent work.

Surfaces have two dimensions — length and width.

We measure surfaces or areas in square inches, square feet, square yards, square rods, acres, and square miles.

A square inch is a square 1 inch long and 1 inch wide.

1. A square foot is a square —— long and —— wide.

2. A square 1 foot long is 12 inches long and 12 inches wide. It contains 12 times 12 square inches, or —— square inches.

3. A square yard is a square —— long and —— wide. It is equal to a square —— feet long and —— feet wide. Its area is —— square feet.

4. $30\frac{1}{4}$ square yards make a square rod. How many square feet make a square rod?

160 square rods make 1 acre.

5. Learn:

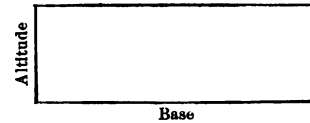
TABLE OF SQUARE OR SURFACE MEASURE

144	square inches (sq. in.)	= 1 square foot (sq. ft.)
9	square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$	square yards	
	or	= 1 square rod (sq. rd.)
$272\frac{1}{4}$	square feet	
160	square rods	= 1 acre (A.)
640	acres	= 1 square mile (sq. mi.)

The side on which a figure seems to stand is its *base.*

The height of a figure from the base is its *altitude.*

6. How many sides has a rectangle?



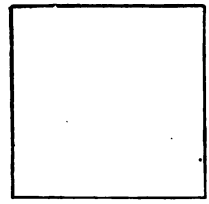
A RECTANGLE

7. How many corners or angles has a rectangle? How do they compare in size?

Each of the angles of a rectangle is a *right angle.*

A figure bounded by four straight lines and having four equal angles is a *rectangle.*

8. How many sides has a square? How do they compare in length?



A SQUARE

9. How many angles has a square? How do they compare in size?

A figure bounded by four equal straight lines and having four equal angles is a *square*.

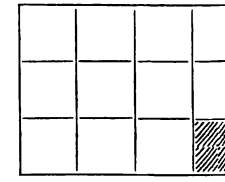
10. In what respects are squares and rectangles alike?

11. In what respect are squares and rectangles unlike?

12. Are all squares rectangles?

The number of square units in the surface of a figure is its *area*.

Let the figure at the right represent a rectangle 4 inches long and 3 inches wide.



The shaded part represents the unit of measurement — one square inch.

13. How many of these units are there in the lower row?

How many rows of these units are there?

Then in the whole figure there are 3 times 4 square inches. The area is 12 square inches.

Note that in finding areas we take these steps:

First. Determine the unit of measurement.

Second. Find the number of these units in one row.

Third. Multiply the number of units in one row by the number of rows.

Think first of the unit of measurement.

The area of a rectangle can always be found by multiplying together its length and its width, when both are expressed in the same unit of measurement (inches, feet, yards, etc.).

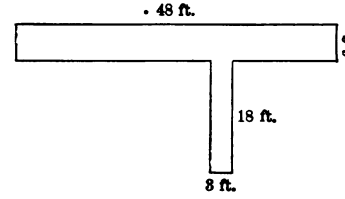
NOTE. Pupils should draw diagrams of rectangles and other plane figures in their problem work until they apprehend the principle involved. The extent to which diagrams are used must be determined by the needs of individuals, since some pupils acquire the powers of visualization and generalization earlier than others.

PROBLEMS *Oral and Written*

1. A post card is 5 inches by 3 inches. How many square inches of writing surface on one side?

2. What is the area of a walk 40 feet long and 3 feet wide?

3. This diagram shows the sidewalk in front of a house and the walk leading to the front door. Find the area of each walk. Express the dimensions of each walk in yards. How many square yards in both walks?



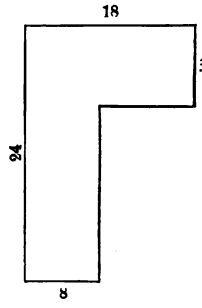
4. A room is 27 feet long and 18 feet wide. Express its dimensions in yards. How many square yards in the floor of the room?

5. A box cover is 15 inches long. Its width is $\frac{2}{3}$ the length. What is the area of the top of the cover?

6. A book is $6\frac{1}{2}$ inches long and 4 inches wide. How much space does it cover on the table?

7. A picture 13 inches by 10 inches is surrounded by a frame 1 inch wide. What are the dimensions of the frame? How much space does the framed picture cover on the wall?

8. The perimeter of a square table is 12 feet. What is the length of one side? What is the area of the top of the table?



9. This diagram represents a field whose dimensions are given in rods. Divide into rectangles and find the area.

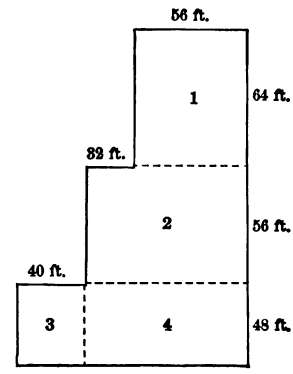
10. Give the dimensions and perimeters of all the different rectangles you can make with 36-inch squares, using all the squares each time. With 48-inch squares. With 60-inch squares.

Find areas and perimeters of these rectangles:

	Length	Width		Length	Width
11.	18 in.	12 in.	**12.**	17 ft.	6 ft.
13.	15 ft.	$8\frac{3}{8}$ ft.	**14.**	14 yd.	16 yd.
15.	$8\frac{3}{4}$ ft.	$2\frac{1}{2}$ ft.	**16.**	25 ft.	5 yd.
17.	$4\frac{1}{2}$ in.	$4\frac{1}{2}$ in.	**18.**	4 yd.	$3\frac{3}{4}$ ft.
19.	30 in.	$2\frac{1}{2}$ ft.	**20.**	$16\frac{1}{2}$ in.	$10\frac{2}{3}$ in.

21. The following diagram represents a plot of ground which was cut up into house lots as indicated by the

dotted lines. Lot 1 was sold for 15¢ a square foot; lot 2 for 12¢ a square foot; lot 3 for 18¢ a square foot; and lot 4 for 20¢ a square foot. Find the selling price of each lot.



22. How many yards of tape will it take to bind a rug $2\frac{1}{2}$ yards long and 1 yard wide? How much space will the rug cover?

Give the areas and the perimeters of:

23. A 4-inch square

24. A 9-yard square

25. A 5-inch square

26. A 10-inch square

27. A 6-foot square

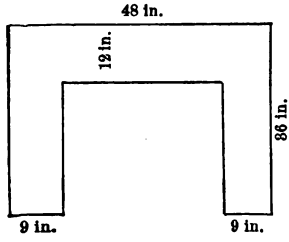
28. An 11-foot square

29. A 7-yard square

30. A 12-yard square

31. An 8-yard square

32. A 20-rod square



33. Find the area of this figure by dividing it into rectangles. Find its perimeter.

34. At 12 cents a square foot, what is the cost of a lot of land 75 feet by 40 feet?

35. A house lot is 75 feet by 48 feet.

(1) Express its dimensions in yards.

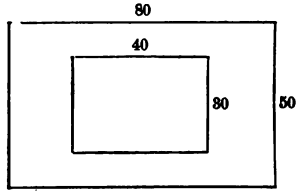
(2) Express the area in square yards. In square feet.

(3) Express the perimeter in feet. In yards.

36. What is the length in yards of a tablecloth that covers 54 square feet if it is 2 yards wide?

37. How many square feet of sod will it take to make a lawn 18 yards long and 9 yards wide?

38. How many strips of turf 4 feet long and 1 foot wide must be used to cover a space 28 feet by 15 feet?



39. A house lot is 50 feet on the street side and has a depth of 80 feet. At 15¢ a foot, what will it cost to fence it? How many square feet in the lot? What is it worth at 18¢ a square foot?

40. A house 30 feet by 40 feet stands in the center of the lot. How far from the street is the front of the house? How far from the sides of the lot does the house stand? How many square feet does the house cover? What part of the lot does it cover?

41. A lot of land is 160 rods long and 1 rod wide. Express its area in square rods. What other name is given to this area?

42. A lot contains 1 acre of land. It is 40 rods long. How wide is it?

43. A farmer has a field containing 2000 square rods. How many acres in the field?

44. It takes 80 rods of fence to inclose a square field. How many acres in the field?

45. What is the area in acres of a square park $\frac{1}{2}$ of a mile on each side?

46. The distance around a square field is $\frac{1}{2}$ of a mile. How many acres in the field?

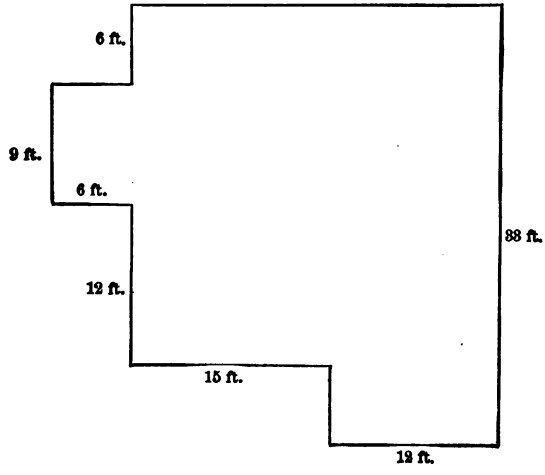
47. How many acres in a lot $\frac{1}{2}$ of a mile long and $\frac{1}{4}$ of a mile wide?

48. At $45 an acre, what is a field 40 rods by 20 rods worth?

49. A lot of land 9 rods by 6 rods was sold for $1188. What was the price per square rod? Per acre?

50. At 50 cents a square yard, what will it cost to lay a sidewalk 60 feet by 6 feet?

51. The following diagram represents the ground plan of a house. Find its perimeter. Find its area.



Express:

52. 2880 square inches as square feet.

53. 2880 square feet as square yards.

54. 2880 square feet as square inches.

55. 2880 square yards as square feet.

56. 2880 square rods as acres.

57. 2880 acres as square rods.

DRAWING TO SCALE *Oral and Written*

1. Draw a line 4 inches long. Divide it into four equal parts. If 1 inch represents 1 foot, how many feet does the line represent? $\frac{1}{2}$ of the line? $\frac{3}{4}$ of the line?

2. If 1 inch represents 2 feet, how many feet does the line stand for? $\frac{1}{2}$ of the line? $\frac{3}{4}$ of the line? A line twice as long?

3. On a map a street is represented by a line 12 inches long. If 1 inch represents 1 rod, how long is the street?

4. Letting 1 inch stand for 5 feet, draw a line that will represent 15 feet. How many inches long is your line?

This is drawing to a scale. The scale you have just used is 1 inch to 5 feet.

Scales on which plans, maps, or diagrams are made are usually indicated in this way: $1'' = 5'$, the sign $''$ meaning inches and the sign $'$ feet. Scale $1'' = 5'$ means that 1 inch represents 5 feet.

5. If on a map a line 1 inch long represents the distance from New York to Philadelphia — 90 miles — what is the scale?

6. From New York to Albany is 140 miles. On the scale $1'' = 14$ miles, how long a line will represent the distance between these two cities?

7. On a scale of 1 inch to 4 feet draw a line that will represent 12 feet.

8. On a scale of 1 inch to 3 feet, how many feet does a line 9 inches long represent?

9. Draw a 4-foot square on a scale of 1 inch to 2 feet.

10. On the scale $1'' = 3'$, what length of lines must you draw to represent a square 1 yard long? A rectangle 12 feet by 9 feet?

11. What is the scale when 3 inches stands for 18 rods?

12. On a map a street 60 rods long is represented by a line 10 inches long. What is the scale?

13. My desk is 5 feet long and 3 feet wide. Draw a picture or diagram of its top, letting 1 inch represent 1 foot.

(1) How many inches long is your diagram?

(2) How many inches wide?

(3) What is the perimeter of the diagram? How many feet does it represent?

(4) What is the area of the diagram? How many square feet does it represent?

14. A flower bed is 60 inches by 40 inches. Draw a plan of it on a scale of 1 inch to 10 inches.

15. Another flower bed is 6 yards square. Draw a plan on a scale of 1 inch to 2 yards.

16. On the scale $1'' = 4'$ draw the diagram of a blackboard 4 feet wide and 24 feet long.

17. What are the dimensions of a room represented by a diagram 8 inches long and 5 inches wide if the scale is 1 inch to 2 feet? What is the floor area?

18. On a builder's plan, drawn to scale 1 foot = 10 feet, a house is represented by a rectangle 4 feet by 3 feet. What are the dimensions of the house? Its area?

19. A dining room is $16'$ by $12'$. Draw diagram to scale $1'' = 4'$.

20. The dining room table is $8'$ by $4'$. Draw a diagram of it in the diagram of the room.

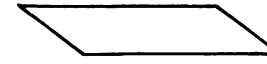
21. Letting 1 inch stand for 20 inches, draw the diagram of a window 60 inches high and 40 inches wide.

22. On a scale of 1 inch to 15 inches draw the diagram of a window sash having 4 panes of glass, each 30 inches by 15 inches.

PARALLELOGRAMS *Oral and Written*

Lines that run in the same direction are *parallel lines*.

A four-sided figure whose opposite sides are parallel is a *parallelogram*.



A PARALLELOGRAM

1. If the shaded part of figure 1 is cut off and placed in the position indicated by the dotted lines, what kind of a figure will you have? See figure 2.

FIG. 1

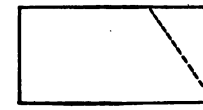


FIG. 2

2. How does the base of the parallelogram compare with the base of the rectangle?

3. How does the altitude of the parallelogram compare with the altitude of the rectangle?

4. Compare the areas of the parallelogram and the rectangle.

5. Draw on paper a parallelogram 3 inches long and 2 inches wide. Cut it out. Cut the parallelogram into two pieces and arrange them to make a rectangle. Compare bases, altitudes, and areas of the parallelogram and rectangle.

6. Draw other parallelograms. Cut, and arrange the parts until you see that a parallelogram is equal to a rectangle having the same base and the same altitude as the parallelogram.

7. How can you find the area of a parallelogram?

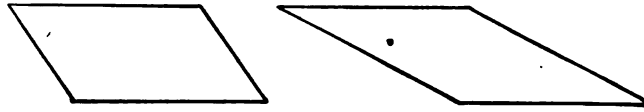
To find the area of a parallelogram, we find the product of its base and its altitude.

8. Draw a rectangle $3\frac{1}{2}$ inches long and 2 inches high. Write the area in the rectangle.

9. Draw a parallelogram whose base is $3\frac{1}{2}$ inches and whose altitude is 2 inches. Write the area in the parallelogram.

10. How do the areas of the two figures you have just drawn compare?

11. Compare the bases and the altitudes of these parallelograms:



12. Find and compare their areas.

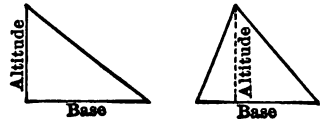
Find areas of parallelograms of these dimensions:

	Base	Altitude		Base	Altitude
13.	12 inches	8 inches	**14.**	18 inches	5 inches
15.	9 feet	10 feet	**16.**	8 yards	9 yards
17.	12 inches	$8\frac{1}{2}$ inches	**18.**	16 feet	$5\frac{3}{8}$ feet
19.	$10\frac{3}{8}$ yards	6 yards	**20.**	$16\frac{1}{2}$ feet	12 feet
21.	4 yards	8 feet	**22.**	18 inches	3 feet

NOTE. Measurement of plane figures made from or drawn on cardboard will prove helpful and interesting. A variety of these figures should be prepared by the teacher, numbered consecutively, and a record of their dimensions and areas kept to facilitate the checking of pupils' work.

Finding measurements and areas of plane figures from the actual figures and from diagrams drawn on the blackboard should precede finding of areas from data given by the teacher.

TRIANGLES *Oral and Written*



A three-sided figure is a *triangle.*

The height of a triangle is its *altitude.*

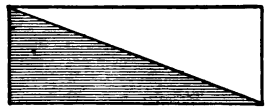


FIG. 1

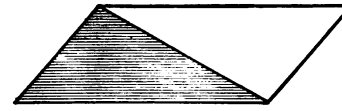


FIG. 2

1. What kind of a figure is the shaded part of figure 1?

2. Compare the base of the triangle with the base of the rectangle.

3. Compare the altitude of the triangle with the altitude of the rectangle.

4. What part of the area of the rectangle is the area of the triangle?

5. How, then, can the area of a triangle be found?

6. In like manner compare the shaded part of figure 2 with the whole parallelogram.

7. Draw on paper a rectangle 4 inches by 3 inches. Cut it into two parts as in figure 1. Compare areas.

8. Draw on paper other parallelograms. Cut each into two parts along the diagonal. Compare areas.

9. Draw on paper a triangle whose base is 4 inches and whose altitude is 2 inches. Cut it out. Cut another triangle exactly like this. Arrange the two triangles so as to form a parallelogram. What are the dimensions of the parallelogram? Compare the base of the parallelogram and the base of the triangles. Compare the altitude of the parallelogram and the altitude of the triangles. What is the area of the parallelogram? The area of each triangle is what part of the area of the parallelogram? What is the area of each triangle?

The area of a triangle is equal to one half the product of its base and its altitude.

To find the area of a triangle, we find one half the product of its base and its altitude.

Note that the dimensions must be expressed in like units.

NOTE. Work like the above should be continued until pupils grasp the principle involved. Different pupils should draw, cut, and compare parallelograms and triangles of different dimensions. Care should be taken that most of the parallelograms and triangles are not rectangles and right-angled triangles.

Give the areas of these triangles :

	ALTITUDE	BASE		ALTITUDE	BASE
10.	10 inches	12 inches	**11.**	15 inches	18 inches
12.	9 inches	3 inches	**13.**	11 inches	7 inches
14.	25 feet	18 feet	**15.**	17 feet	12 feet
16.	13 rods	8 rods	**17.**	7 yards	9 yards

18. What are the base and altitude of the largest triangle you can cut from a piece of paper 4 inches square?

19. What are the dimensions of the largest triangle you can cut from a piece of paper 5 inches by 3 inches?

20. What are the base and altitude of the largest triangle you can draw on a sheet of your arithmetic paper? How does the area of this triangle compare with the area of the sheet on which it is drawn?

21. In the corner of a room is a triangular shelf. The two sides that touch the wall are each 10 inches in length. What is the area of the shelf? On the shelf stands a box

4 inches long and $2\frac{1}{2}$ inches wide. How many square inches of the shelf does it cover?

22. Three roads form the sides of a triangular lot. The base of the lot is 22 feet and the altitude is 18 feet. How many square feet in the lot?

23. A field 32 rods long and 20 rods wide is separated into two equal triangular parts by a path running between two opposite corners. What are the base and the altitude of each part? How many acres in each part?

24. At 15 cents a square foot, what is the value of a three-sided lot of land whose base is 64 feet and whose altitude is 40 feet?

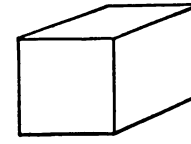
25. How many square yards are there in a triangular lot whose base is 18 yards and whose altitude is one half the length of the base?

26. The height of a triangle is 24 inches. The base is $\frac{3}{4}$ as long. What is the area of the triangle?

27. A triangular flower bed is 36 inches on each side. How many feet of wire netting will inclose it?

MEASURING VOLUMES *Oral and Written*

A number of 1-inch cubes should be used in teaching this subject.



1. How many sides or faces has a cube?

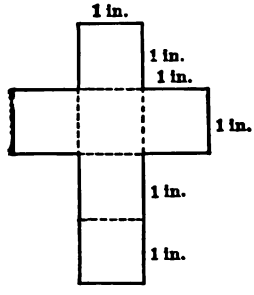
2. How do the sides compare in shape?

3. How do the sides compare in size?

A solid bounded by six equal sides or faces is a *cube.*

A solid has three dimensions—length, breadth, and thickness.

4. Draw on cardboard a figure like this. Cut it out and fold on dotted lines. Paste, sew, or pin the edges together. You have made a cube 1 inch long, 1 inch wide, and 1 inch high. This is called an inch cube or a cubic inch. How many sides or faces has it? How do they compare in size? What is the shape of each face? What is the area of each face? What is the area of all the faces?



5. Could you have told the area of the surface of the cube from the diagram?

6. With the inch cubes build a solid like figure 1, 3 inches long, 2 inches wide, and 1 inch thick. This is a rectangular solid or rectangular prism.

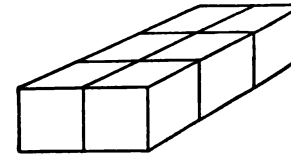


FIG. 1

How many cubic inches are there in 1 row? In both rows?

We say its contents or volume is 6 cubic inches.

The number of cubic units in a solid is its *volume.*

7. With the inch cubes build a solid like figure 2, 3 inches long, 2 inches wide, and 3 inches high.

How many cubic inches in 1 row of the bottom layer? Then in the bottom layer there are 2 times 3 cubic inches, or 6 cubic inches.

How many layers are there? Then in the whole solid there are 3 times 6 cubic inches, or 18 cubic inches.

Note that in finding volumes we take these four steps:

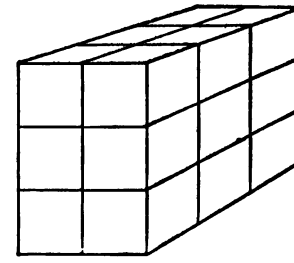


FIG. 2

First. Determine the unit of measurement.

Second. Find the number of these units in one row of the lower layer.

Third. Multiply the number of units in one row by the number of rows.

Fourth. Multiply the number of units in one layer by the number of layers.

Think first of the unit of measurement.

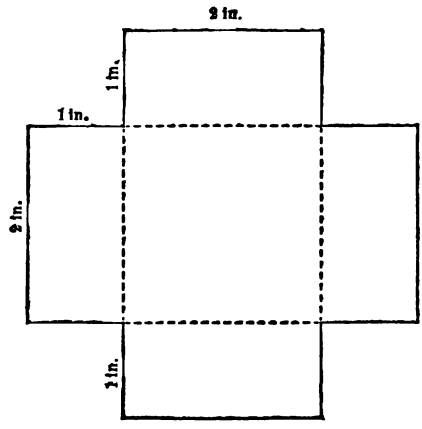
The volume of a solid can always be found by multiplying together its length, its width, and its height, when all are expressed in the same unit of measurement (inches, feet, yards, etc.).

NOTE. Practice in computing volumes of blocks, boxes, and so forth, from measurements made by pupils, should precede the solution of problems from data given by the teacher.

Give the volumes of these rectangular prisms:

8. 2 in. by 4 in. by 5 in.
9. 3 in. by 4 in. by 2 in.
10. 4 in. by 5 in. by 3 in.
11. 6 in. by 5 in. by 2 in.
12. 5 in. by 8 in. by 2 in.
13. 3 in. by 8 in. by 2 in.
14. 3 in. by 5 in. by 4 in.
15. 6 in. by 2 in. by 8 in.
16. 10 in. by 3 in. by 6 in.
17. 12 in. by 5 in. by 4 in.

18. With the help of this diagram construct a box that will hold 4 cubic inches.



19. How high will you have to make the sides of a box of the same base to hold twice as much? Make one.

20. With the diagram below as an aid, construct a rectangular prism 3 inches by 2 inches by 2 inches.

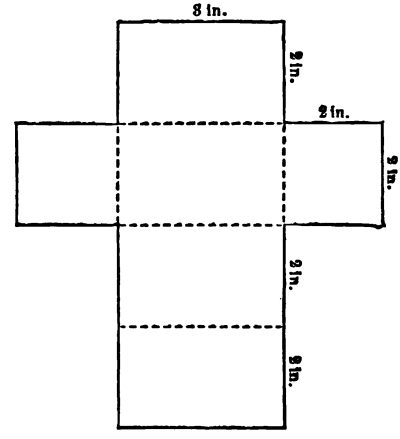
21. How many sides has this rectangular prism?

22. Of what shape are the sides? Are all the sides equal?

23. What is the area of the two ends? Of the four sides? What is the total area of the six sides?

24. Could you determine the surface area from the pattern?

25. How many 1-inch prisms could you put into the prism you have just made?



26. A cube 1 foot long, 1 foot wide, and 1 foot high is a cubic foot.

27. Express its dimensions in inches.

28. A cubic foot contains $12 \times 12 \times 12$ cubic inches, or —— cubic inches.

29. Describe a cubic yard.

30. Express its dimensions in feet.

31. What is its volume in cubic feet?

32. Write the table of cubic measure.

33. Make a pattern of a 2-inch cube. Cut it out and fold it into a cube.

How long is this cube? How wide? How high?

What is the area of one of its faces? Of all its faces?

What is its volume? How many 1-inch cubes will it take to make a 2-inch cube?

34. What is the volume of a 3-inch cube?

35. How many inch cubes can you put into a box 4 inches on each edge?

36. How many cubic inches are occupied by a book 6 inches long, $3\frac{1}{2}$ inches wide, and 1 inch thick?

37. The inside measurements of a box are 5 inches, 3 inches, $1\frac{1}{2}$ inches. What is its capacity?

38. A drawer in a desk is 8 inches by 5 inches by $1\frac{3}{4}$ inches. What is its capacity?

39. A coal bin is 10 feet by 6 feet by 4 feet. How many cubic feet of coal will it hold when even full?

Find the volumes of these rectangular prisms:

	Length	Width	Height		Length	Width	Height
40.	10 ft.	6 ft.	5 ft.	41.	9 in.	7 in.	4 in.
42.	15 ft.	12 ft.	9 ft.	43.	18 in.	15 in.	1 ft.
44.	14 ft.	4 ft.	$\frac{3}{4}$ ft.	45.	20 in.	$8\frac{1}{2}$ in.	5 in.
46.	$18\frac{3}{4}$ ft.	16 ft.	$4\frac{1}{2}$ ft.	47.	27 in.	$4\frac{2}{3}$ in.	$\frac{1}{2}$ ft.
48.	$3\frac{5}{8}$ in.	3 in.	$1\frac{1}{3}$ ft.	49.	$1\frac{1}{4}$ ft.	1 ft.	18 in.

50. Find the surface areas of the prisms in examples 8 to 17 on page 311.

WOOD MEASURE

Wood is usually sold by the cord.

1. A pile of wood 8 ft. by 4 ft. by 4 ft. is a cord. How many cubic feet in a cord?

2. $\frac{1}{8}$ of a cord is a cord foot. How many cubic feet in a cord foot?

3. Learn:

16 cubic feet = 1 cord foot (cd. ft.)
8 cord feet = 1 cord (cd.)
128 cubic feet = 1 cord

4. How many cords of wood in a pile 8 feet long, 4 feet wide, and 8 feet high? How many in a pile 16 feet by 4 feet by 8 feet?

5. A wagon body 4 feet wide and 12 feet long has on it a pile of wood 6 feet high. How many cords?

6. By the roadside near a farmer's house I saw a pile of wood 4 feet wide, 6 feet high, and 18 feet long. How many cords in the pile?

7. A leather firm bought from this farmer a pile of hemlock bark 4 ft. × 4 ft. × 16 ft. How many cords?

8. Express 1 cord, 16 cord feet as cords.

9. How many cubic feet in three quarters of a cord?

10. How many cords in 1728 cubic feet of bark?

DECIMALS *Oral and Written*

Dimes, cents, and mills are decimal parts of a dollar.

Dimes are written in the first place at the right of the decimal point as *tenths* of a dollar; cents are written in the second place at the right as *hundredths* of a dollar; mills in the third place at the right as *thousandths* of a dollar.

A dime, or a tenth of a dollar, is written \$.1.

A cent, or a hundredth of a dollar, is written \$.01.

A mill, or a thousandth of a dollar, is written \$.001.

\$0.87 may be read 87 hundredths of a dollar.

\$0.875 may be read 875 thousandths of a dollar.

Read as parts of a dollar:

1. \$0.6; \$0.05; \$0.003; \$0.802; \$0.025.

2. What do the 0's show in the numbers you have just read?

3. Write decimally $\frac{5}{10}$ of a dollar; $\frac{50}{100}$ of a dollar; $\frac{37}{100}$ of a dollar; $\frac{125}{1000}$ of a dollar; $\frac{25}{1000}$ of a dollar; $\frac{5}{1000}$ of a dollar.

Write decimally:

4. 7 hundredths of a dollar.

5. 70 hundredths of a dollar.

6. 75 hundredths of a dollar.

7. 75 thousandths of a dollar.

8. 225 thousandths of a dollar.

9. 5 thousandths of a dollar.

10. How many places are used to express tenths ot a dollar? Hundredths of a dollar? Thousandths of a dollar?

We can express other things besides dimes, cents, and mills as tenths, hundredths, and thousandths. Thus, .25 yd. This means 25 hundredths of a yard.

11. Read: .5 bu.; .75 A.; .287 mi.; .08 rd.

2.25 yd. means 2 whole yards and 25 hundredths of a yard. It is read two and twenty-five hundredths yards.

Whenever we read a number made up of a whole number and a decimal, we always use the word *and* to mark the decimal point.

12. Read: 2.5 ft.; 3.275 mi.; 4.08 sq. rd.; 7.006 A.

13. Write decimally:

$\frac{7}{10}$; $\frac{8}{100}$; $\frac{47}{100}$; $\frac{84}{1000}$; $\frac{6}{1000}$;

$\frac{75}{100}$ bu.; $\frac{5}{10}$ in.; $\frac{22}{1000}$ A.; $\frac{205}{1000}$ mi.

14. Write decimally:

$5\frac{3}{10}$; $2\frac{4}{100}$; $3\frac{15}{100}$; $6\frac{2}{1000}$;

$3\frac{7}{10}$ yd.; $7\frac{25}{100}$ in.; $5\frac{8}{100}$ sq. yd.; $8\frac{4}{1000}$ mi.

Read:

15.	.8	.96	.07	.519	.806	.087	.005
16.	.3	.03	.33	.303	.033	.003	.333
17.	4.7	3.64	5.07	7.602	8.319	9.054	2.008

Write in figures:

18. Seven tenths. **19.** Five hundredths.

20. Nine thousandths. **21.** Seventeen thousandths.

22. Sixty-eight hundredths.

23. One hundred two thousandths.

24. Three hundred eighty-seven thousandths.

25. Four and nineteen hundredths.

26. Thirty-two and four hundred seven thousandths.

27. Sixteen and six thousandths.

CHANGING DECIMALS TO COMMON FRACTIONS

1. Write as common fractions: .1; .01; .001.

$$.1 = \tfrac{1}{10} \quad .01 = \tfrac{1}{100} \quad .001 = \tfrac{1}{1000}$$

Write these decimals as common fractions:

2.	.2	.4	.6	.8	.3	.5	.7	.9
3.	.12	.07	.67	.05	.83	.07	.56	.03
4.	.125	.402	.019	.009	.047	.004	.103	.005

5. Write .6 as a common fraction and change to its simplest form:

$$.6 = \frac{6}{10} \div \frac{2}{2} = \frac{3}{5}$$

Express these decimals as common fractions in their simplest form:

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
6.	.2	.4	.6	.8	.02	.04	.06	.08
7.	.25	.50	.75	.66	.32	.56	.24	.48
8.	.15	.45	.65	.35	.85	.64	.84	.76

Express as common fractions in their lowest terms:

.5 .50 .500

How do .5, .50, and .500 compare in value?

Ciphers annexed to a decimal do not change its value. Why?

CHANGING COMMON FRACTIONS TO DECIMALS

1. Write as tenths of a dollar: $\frac{1}{2}$ of a dollar; $\frac{1}{5}$ of a dollar; $\frac{3}{5}$ of a dollar; $\frac{3}{5}$ of a dollar; $\frac{4}{5}$ of a dollar.

2. Write as hundredths of a dollar: $\frac{1}{2}$ of a dollar; $\frac{1}{4}$ of a dollar; $\frac{3}{4}$ of a dollar; $\frac{1}{5}$ of a dollar; $\frac{2}{5}$ of a dollar; $\frac{3}{5}$ of a dollar; $\frac{4}{5}$ of a dollar.

3. Write as hundredths of a dollar: $\frac{1}{20}$ of a dollar; $\frac{7}{20}$ of a dollar; $\frac{1}{25}$ of a dollar; $\frac{12}{25}$ of a dollar; $\frac{1}{50}$ of a dollar; $\frac{9}{50}$ of a dollar.

4. Express decimally, first as tenths, then as hundredths: $\frac{1}{2}$; $\frac{1}{5}$; $\frac{3}{5}$; $\frac{2}{5}$; $\frac{4}{5}$.

5. Express decimally as hundredths: $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{10}$; $\frac{3}{10}$; $\frac{7}{10}$; $\frac{9}{10}$; $\frac{1}{20}$; $\frac{11}{20}$; $\frac{1}{25}$; $\frac{6}{25}$; $\frac{9}{25}$; $\frac{1}{50}$; $\frac{7}{50}$; $\frac{27}{50}$.

Write as whole numbers and decimals:

6. $2\frac{1}{2}$; $2\frac{1}{4}$; $5\frac{3}{4}$; $2\frac{1}{5}$; $1\frac{3}{5}$; $7\frac{3}{10}$.

7. $5\frac{1}{20}$; $3\frac{7}{20}$; $6\frac{1}{25}$; $2\frac{4}{25}$; $7\frac{1}{50}$; $5\frac{17}{50}$.

8. Express as the decimal of a foot: 6 inches; 3 inches; 9 inches.

9. What decimal part of an hour is 3 minutes?

3 minutes $= \frac{3}{60} = \frac{1}{20} = \frac{5}{100} = .05$ of an hour.

Express as decimals of an hour:

10. 30 minutes; 15 minutes; 45 minutes.

11. 12 minutes; 24 minutes; 36 minutes; 48 minutes.

12. 6 minutes; 18 minutes; 42 minutes; 54 minutes.

13. 9 minutes; 21 minutes; 33 minutes; 57 minutes.

DECIMALS: ADDITION *Oral and Written*

Add:

1. \$0.60 / .20

2. 6 dimes / 2 dimes

3. 6 tenths / 2 tenths

4. .6 / .2

5. \$0.05 / .04

6. 5 cents / 4 cents

7. 5 hundredths / 4 hundredths

8. .05 / .04

9. \$0.375 / .233

10. 375 thousandths / 233 thousandths

11. .375 / .233

In adding decimals, why must tenths come under tenths, hundredths under hundredths, and so on?

Add by rows and by columns:

	A	*B*	*C*	*D*	*E*
12.	6.73	+ 18.4	+ 8.5	+ 60	+ 4.003
13.	5.8	+ 7.29	+ 7.06	+ 6	+ .87
14.	.97	+ 3.07	+ 4.12	+ .6	+ .008
15.	.008	+ 15.007	+ 10.01	+ .06	+ 5.17
16.	70.49	+ 3.9	+ 9.004	+ .006	+ 4.09

Write in columns and add:

17. .5, .27, .08, .762, .007.
18. .007, .64, .303, .09, .3.
19. .606, .04, .005, .008, .7, .3.
20. .302, .08, .009, .54, .16, .016.
21. .97, .087, .07, .05, .09, .008.
22. .07, .017, .009, .108, .05, .012.
23. 4.37, 2.05, 9.007, .03, 4.1.
24. 8.007, .37, 6.09, 4.304, .006.
25. 5.5, .004, 3.018, 6.704, .076.
26. 4.85, 3.001, 5.07, .008, .02.

DECIMALS: SUBTRACTION *Oral and Written*

Subtract:

1. \$0.63	2. \$0.80	3. \$0.08	4. \$0.40
.44	.60	.05	.37
5. \$1.00	6. \$0.625	7. \$0.600	8. \$0.008
.05	.375	.045	.005
9. 1.000	10. 1.000	11. .087	12. .308
.025	.004	.009	.088
13. .402	14. .072	15. .067	16. .6
.391	.006	.059	.27

In example 16 think .6 as hundredths.

17. .8	**18.** .57	**19.** .7	**20.** .69	**21.** .1
.34	.3	.07	.6	.05
22. .563	**23.** .427	**24.** .8	**25.** .5	**26.** .3
.5	.42	.425	.463	.292

27. From .4 take .4 ; .04 ; .004.

28. From 8 take .8 ; .08 ; .008.

29. From one take one tenth ; one hundredth ; one thousandth.

30. From one tenth take one tenth ; one hundredth ; one thousandth.

31. From one hundredth take one hundredth ; one thousandth.

32. From ten take one tenth.

33. From one hundred take one hundredth.

34. From one thousand take one thousandth.

35. From 13.7 take 6.08.

36. Take .017 from 6.6.

37. From 1.672 take 1.005.

38. Take .305 from 1.055.

39. From 27.06 take 14.3.

40. Take 14.07 from 70.04.

41. From 3.002 take .998.

42. Take 7.006 from 10.04.

43. From 643.7 take .691.

44. Take 1.125 from 11.325.

PROBLEMS *Oral and Written*

1. Edward walked .3 of a mile and rode .5 of a mile. How far did he go?

2. A stick of braid contained 4 yards. The dressmaker used .5 of a yard. How much was left?

3. In making candy, Emma used .25 of a pound of chocolate and .75 of a pound of sugar. What was the weight of both?

4. If you spend .6 of your money, how much will you have left?

5. Charles bought a necktie for .25 of a dollar and a collar for .15 of a dollar. What part of a dollar did he pay for both?

6. My pencil was 7 inches long. How long was it after I had used 1.75 inches?

7. The grocer sold .5 of a bushel of potatoes to one customer and .625 of a bushel to another. How many bushels did he sell?

8. Mr. Hollis has three pastures, one of 4.75 acres, one of 25.5 acres, and one of 8.42 acres. What is the area of the three?

9. William raised a bushel of strawberries. He sold .125 of a bushel to Mrs. Waite, .25 of a bushel to Mrs. Long, and the rest to the grocer. What part of a bushel did he sell to the grocer?

10. The three sides of a triangle are 12.4 ft., 18.65 ft., and 15.75 ft. What is the perimeter?

11. The perimeter of a triangle is 8.5 ft. Two sides are respectively 2.25 ft. and 3.8 ft. What is the length of the third side?

12. A playground contains 7.32 acres. In it is a pond covering 1.67 acres. What is the area not covered by the pond?

13. A tub of maple sugar weighs 34.625 pounds. The tub itself weighs 3.875 pounds. What is the weight of the sugar?

14. The weights of five tubs of butter were 30.125 lb., 28.5 lb., 29.875 lb., 30.25 lb., and 27 lb. What was the total weight?

DECIMALS: MULTIPLICATION *Oral and Written*

1. 3 times 3 apples = ——
2. 3 times 3 tenths = ——

3. $3 \times \frac{3}{10} = \frac{9}{10} = .9$
4. $3 \times .3 = .9$

5. Multiply .3 by .3.

$\frac{3}{10} \times \frac{3}{10} = \frac{9}{100} = .09$

$.3 \times .3 = .09$

Express both decimals as common fractions.

Multiplying $\frac{3}{10}$ by $\frac{3}{10}$, we get $\frac{9}{100}$, which, written decimally, is .09.

In multiplying .3 by .3 it is clear that, since the denominators are 10 and 10, the denominator of the product must be 10×10, or 100. A decimal expressing hundredths occupies two decimal places, which is the sum of the decimal places in the multiplicand (.3) and the multiplier (.3).

6. Multiply .03 by .3.

$\frac{3}{100} \times \frac{3}{10} = \frac{9}{1000} = .009$

$$\begin{array}{r} .03 \\ .3 \\ \hline .009 \end{array}$$

The product of the denominators is 1000. A decimal expressing thousandths occupies three decimal places. This is the sum of the decimal places in the multiplicand (.03) and the multiplier (.3).

To multiply decimals, we multiply as in whole numbers, and point off as many decimal places in the product as there are decimal places in both multiplicand and multiplier.

Note that the "pointing off" is the multiplying together of the denominators.

7. How many decimal places are there in the product when we multiply units and tenths? $3 \times .2$.

8. How many when we multiply units and hundredths? $3 \times .02$.

9. How many when we multiply units and thousandths? $3 \times .002$.

10. How many when we multiply tenths and tenths? $.3 \times .2$.

11. How many when we multiply tenths and hundredths? $.3 \times .02$.

Multiply, orally, by 2 each number in the table:

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
12.	1	4	7	2	6	9	3	5	8
13.	.1	.4	.7	.2	.6	.9	.3	.5	.8
14.	.01	.04	.07	.02	.06	.09	.03	.05	.08

15. Use 1, 3, 5, 7, 9, 2, 4, 6, 8, 10, 11, 12 as multipliers.

16. Use .1, .3, .5, .7, .9, .2, .4, .6, .8, 1.1, 1.2 as multipliers.

17. Multiply each number in the first two rows by .01, .02, .03, .04, .05, .06, .07, .08, .09.

18. Victor is 7.5 years old and Hubert is twice as old. How old is Hubert?

19. Sarah has 50 cents. Marion has .5 as much. How many cents has Marion?

20. There are 80 trees in an orchard. .3 of them are pear trees. How many pear trees?

21. What is the area of a square .5 of a yard long? What is its perimeter?

22. The three sides of a triangle are each 2.4 feet long. What is the total length of the sides?

23. How many square rods in a rectangle .7 of a rod long and .6 of rod wide? What is the perimeter?

Multiply:

24. .76 × 42	**25.** 2.07 × 63	**26.** 3.4 × 8.7	**27.** 6.25 × 1.4	**28.** 3.07 × 8.9
29. 39 × .07	**30.** .045 × 52	**31.** 27.3 × 4.4	**32.** 78.5 × .08	**33.** 4.55 × 6.6
34. .096 × 75	**35.** .308 × 47	**36.** 408 × .027	**37.** 78.5 × 1.07	**38.** .875 × 64
39. 25.04 × 56	**40.** 500.5 × 3.17	**41.** 648 × .035	**42.** 39.3 × 2.06	**43.** 720 × .225
44. 30.05 × .48	**45.** 520.7 × 4.05	**46.** 28.76 × 1.15	**47.** 1728 × .375	**48.** 17.28 × 37.5

49. The multiplicand is .643; the multiplier is 867; what is the product?

50. Multiply sixty and six tenths by ten and one tenth.

51. $6.06 \times 5.5 \times 2.002 = ?$

MULTIPLYING BY 10, 100, 1000

$.222 \times 10 = 2.22$

$.222 \times 100 = 22.2$

$.222 \times 1000 = 222.$

1. In multiplying .222 by 10, the decimal point was moved how many places to the right? How many places to the right was it moved in multiplying by 100? How many places to the right was it moved in multiplying by 1000?

2. Change the decimal point in 1.234 so that you will have a number 10 times as great. So that you will have a number 100 times as great. So that you will have a number 1000 times as great.

Write numbers 10 times as great as these:

3. .284 3.75 42.6 .008 .06 .3

4. 3.706 .903 4.62 .05 .4 .007

5. Write numbers 100 times as great; 1000 times as great.

6. What is the weight of 10 chickens if each weighs 3.75 pounds?

7. What is the total length of 100 boards, each 6.25 feet long?

8. How many yards of cloth in 1000 pieces, each of which contains 27.5 yards?

9. Frederick's cap cost $0.25; his shoes cost 10 times as much, and his suit 100 times as much. How much did the shoes cost? The suit? How much did all cost?

MULTIPLYING BY .1, .01, .001

$$222 \times .1 = 22.2$$
$$222 \times .01 = 2.22$$
$$222 \times .001 = .222$$

1. In multiplying 222 by .1 the decimal point was moved how many places to the left? How many in multiplying by .01? How many in multiplying by .001?

2. Change the decimal point in 3456 so that you will have a number .1 as great. So that you will have a number .01 as great. So that you will have a number .001 as great.

Multiply by .1:

3.	$525	$37.50	$2.45	$0.70	$0.66	$0.04
4.	236	42.5	3.17	.5	.75	.03

Multiply by .01:

5.	4325	372.5	30.4	100.1	.2	1.1

Multiply by .001:

6.	46,800	1000	144	36	8	1

7. Howard had 80 marbles. He lost .1 of them. How many did he lose?

8. Mr. Wilson paid $1000 for an automobile and .01 as much for a license to run it. How much did he pay for the license?

9. Out of 125,000 yards of cloth .001 was found imperfect. How many yards were poor?

PROBLEMS *Written*

1. There are 16.5 feet in a rod. What is the length in feet of a fence 7 rods long?

2. How many feet in 320 rods or 1 mile?

3. What is the area of a square 15.4 yards long?

4. Henry has 8 rows of peas. He gathers 2.75 bushels from 1 row. How many bushels will he probably get from the other rows?

5. Mr. Moulton mowed 2.6 acres of grass in a day. How many acres will he mow in 3.5 days?

6. A square lot is 32.07 rods on a side. How many rods of wall will inclose it?

7. A cubic foot of water weighs 62.5 pounds. What weight of water will a tank 2 feet square and 3 feet high hold?

8. Ice weighs .92 as much as water. What is the weight of a cubic foot of ice?

9. How far will a railroad train run in 2.4 hours if the rate is 40.75 miles an hour?

10. A cow gives 3.2 gallons of milk a day. How many pounds is this if a gallon weighs 8.625 pounds?

11. Mr. Slater's house-lot contains .65 of an acre. His pasture is 10 times as large, and his garden is .1 as large. What is the size of the pasture? Of the garden?

12. Our school paid 75 dollars for trees for the school grounds, .1 as much for flowering shrubs, and .01 as much for seeds for the vegetable garden. How much was paid for shrubs? For seeds? How much was paid for all?

DECIMALS: DIVISION *Oral and Written*

Divide:

1. 2)8 dollars
2. 2)$8.00
3. 2)$1.68
4. 2)$0.68
5. 2)$0.60
6. 2)6 tenths
7. 2).6
8. 2)64 hundredths
9. 2).64
10. 2)648 thousandths
11. 2).648
12. 2).608
13. 2).812

Note that in dividing a decimal by a whole number the decimal point in the quotient comes directly under the decimal point in the dividend. The first step in division is to write the decimal point in the dividend.

Divide, and test your work:

	A	*B*	*C*	*D*	*E*
14.	3)3.696	3)36.96	3)3.696	3).603	3).906
15.	4)3.08	4)177.2	4)1.984	4)2.24	4)4.08
16.	5)14.5	5)2.045	5)4.05	5)3.55	5)2.65
17.	6)2.76	6).72	6)8.4	6).84	6).726
18.	7)8.05	7)85.4	7).924	7)285.6	7)35.7
19.	8)1.28	8)34.4	8)4.32	8).808	8)11.52
20.	9)12.78	9)7.2	9)63.36	9)54.72	9)9.009

21. Divide .36 by 9.

9)$\overline{.36}$ There being no tenths in the quotient, we write a 0 in
.04 the tenths' place.

22. Divide .008 by 4.

4)$\overline{.008}$ Why do we write two 0's in the quotient in this divi-
.002 sion?

23. Divide .2 by 5. .2 may be written .20.

Divide, and test your work:

	A	*B*	*C*	*D*	*E*
24.	3)$\overline{.018}$	2)$\overline{.08}$	4)$\overline{.036}$	7)$\overline{.056}$	5)$\overline{.005}$
25.	6)$\overline{.072}$	7)$\overline{.049}$	4)$\overline{.028}$	5)$\overline{.14}$	2)$\overline{.01}$
26.	8)$\overline{.04}$	5)$\overline{.065}$	9)$\overline{.729}$	6)$\overline{.426}$	8)$\overline{.056}$
27.	7)$\overline{.084}$	6)$\overline{.006}$	8)$\overline{.12}$	7)$\overline{.28}$	9)$\overline{.198}$

28. Divide 12.88 by 28.

.47
28)$\overline{13.16}$ In long division be careful to place the decimal
11 2 point in the quotient directly over the decimal point
196 in the dividend.
196

Divide, and test your work:

	A	*B*	*C*	*D*
29.	22.68 by 27	34.68 by 34	9.90 by 45	1.44 by 16
30.	17.92 by 32	5.184 by 24	51.84 by 72	15.75 by 15
31.	17.28 by 36	172.8 by 24	1.728 by 48	34.56 by 12
32.	345.6 by 16	3.456 by 32	34.56 by 54	.3456 by 27
33.	35.68 by 16	775.2 by 19	7.752 by 38	77.52 by 57

Find the quotient of :

34.	4.536 ÷ 42	26.20 ÷ 35	5.12 ÷ 64	21.28 ÷ 76
35.	46.72 ÷ 73	4.672 ÷ 146	.522 ÷ 29	5.04 ÷ 84
36.	74.16 ÷ 72	68.4 ÷ 90	374.48 ÷ 62	17.385 ÷ 57
37.	416.56 ÷ 82	6.916 ÷ 28	38.52 ÷ 36	1297.8 ÷ 63
38.	.552 ÷ 92	3.12 ÷ 39	44.8 ÷ 56	816.08 ÷ 202

PROBLEMS *Written*

1. A coal dealer sent out 5.25 tons of coal in 3 equal loads. What was the weight of each load?

2. Maggie used .5 of a yard of cloth in making 2 dresses for her dolls. How much was used for each dress?

3. A merchant sold 8 pairs of shoes for $13.20. How much was this a pair?

4. What is the side of a square whose perimeter is 36.24 square rods?

5. Richmond rode his bicycle 17.4 miles on Tuesday and $\frac{1}{6}$ as far on Wednesday. How far on Wednesday?

6. It took 15 fence rails to build a fence 118.5 feet long. What was the length of each rail?

7. If 57.75 tons of hay were cut from 7 fields, what was the average cut from each field?

8. My gas bills for six months were $1.89, $2.16, $2.43, $1.80, $2.70, $2.52. What was the average cost of the gas a month?

9. At the rate of 17 miles an hour, how long will it take to go to a place 40.8 miles away?

10. In 6 days a range burned 2.4 thousand cubic feet of gas. What part of a thousand cubic feet was this a day?

BILLS AND RECEIPTS

WASHINGTON, July 1, 1910.

Mr. CHARLES R. WATSON

Bought of CROSBY & MARSH

Mar. 3	3 pr. Shoes	@ $2.15	$6	45		
Apr. 7	3 pr. Slippers	@ .83	2	49		
June 1	2 pr. Rubbers	@ .69	1	38		
	Received payment July 15, 1910 CROSBY & MARSH By Goodwin.				$10	32

When were the above purchases made?

By whom were the goods bought?

From whom were they bought?

What was bought?

What did each kind cost?

What did all cost?

When was the bill paid?

What shows that the bill has been paid?

Was the money paid directly to the owners of the store or to one of their clerks? How do you know?

Who is the creditor in the above bill? Why?

Who is the debtor? Why?

Mr. Ames sells his black horse to Mr. Baker. Who is the debtor?

Mr. Childs buys a house from Mr. Burns. Who is the debtor?

1. Complete the following bill :

BUFFALO, July 29, 1910.

MRS. HENRY P. DUNCAN

Bought of ARTHUR P. DAVIS

2 lb. Figs	@	$0.20	$	
3½ lb. Raisins	@	.14		
5¾ lb. Mixed Nuts	@	.16		
4 lb. Candy	@	.35		
				$
Received payment ARTHUR P. DAVIS.				

When purchases are made at one time, the date is written in the heading only.

Make out bills for the following school supplies. Buyer, the city in which you live. Seller, yourself.

2.

120 reams of paper @ 35¢
12 boxes of pens @ 32¢
25 dozen pencils @ 18¢

3.

50 arithmetics @ 65¢
75 arithmetics @ 42¢
20 number cards @ 3½¢

4.

68 grammars @ 54¢
38 geographies @ 95¢
18 geographies @ 75¢

5.

4 wall maps @ $3.75
100 spelling books @ 18¢
35 readers @ 25¢

6. Make out the bill for 10 grammars, 12 number cards, and 20 spelling books at the prices given above.

7. Mr. George R. Hamilton used 14,000 pounds of ice during the year 1909. Make out his bill at $3 a ton.

8. Mr. Alfred Smith buys $6\frac{1}{2}$ tons of coal at $6.50 a ton and 2 tons at $6.75 a ton. Make out his bill.

9. The pupils in the Jackson school bought the following seeds for their school garden: 8 10-cent packets of nasturtiums, 6 5-cent packets of poppies, and 5 5-cent packets of asters. Make out the bill.

10. Make out your bill for cutting your neighbor's lawn three times: on July 10 you work 6 hours, on July 24 you work $6\frac{1}{2}$ hours, and on Aug. 7, $5\frac{1}{2}$ hours. You receive 20 cents per hour.

11. Imagine that you sell to a hotel 4 barrels of potatoes at $3.35 per barrel, 2 bushels of peas at $1.75 per bushel, 2 boxes of lettuce at 65 cents each, and $1\frac{1}{2}$ bushels of beans at $1.12 per bushel. Make out the bill.

12. Robert put electric bells in his house. He paid $0.75 for one bell and $0.60 for the other. It took $1\frac{1}{2}$ pounds of wire at 20 cents a pound. He used a 6-cent paper of tacks, and 2 buttons at 12 cents each. Make out the bill, using your own name as seller.

13. Make out the bill for three articles purchased by your mother at the grocer's.

14. Make out your milk bill for the month of April.

15. Make out the bill for three kinds of fruit you see every day in the stores.

16. Make out other bills for goods purchased at different stores, using the prices given in the daily paper.

INTERMEDIATE ARITHMETIC

PART II

NOTATION AND NUMERATION

1. How many places are required to write ten in figures? One hundred? One thousand? Ten thousand? One hundred thousand? One million? Ten million? One hundred million?

2. Why is our system of writing numbers called a decimal system?

3. Using the figures 2, 8, 3, 9, 6, write the largest number possible. Read it.

4. Using the same figures, write the smallest number possible. Read it.

5. Write a number occupying five places which has no tens and no hundreds. Read it.

6. Using the figures 3 and 3, and as many 0's as necessary, write:

(1) A number in which the left-hand 3 has ten times the value of the right-hand 3.

(2) A number in which the left-hand 3 has one hundred times the value of the right-hand 3.

(3) A number in which the left-hand 3 has one thousand times the value of the right-hand 3.

7. Compare the values of the 7's in 77; in 707; in 770; in 7007; in 7070; in 70,007.

The value of a figure in a number depends upon two things:

(1) Its value in the scale 1, 2, 3, 4, 5, 6, 7, 8, 9.

(2) Its place in the number.

The figures 1, 2, 3, 4, 5, 6, 7, 8, 9, are called significant figures.

8. Keeping in mind the values of the significant figures, tell how many times the value of the right-hand figure is represented by the left-hand figure in 42; in 930; in 603; in 5010; in 8002.

9. Write a number which has the figure 2 in the units' place, and a figure in the tens' place which in that place expresses forty times as much.

Write in figures:

1. Seven hundred eight.

2. Nineteen thousand forty-two.

3. Sixty thousand five hundred nine.

4. Two hundred forty-six thousand three hundred sixty.

5. Eight hundred ten thousand six hundred forty-nine.

6. Three million one hundred eighty-four thousand two hundred four.

7. Ninety-one million fifty-seven thousand five hundred.

8. Five hundred six million.

9. Four hundred eighteen million twenty-eight thousand seven hundred thirty-two.

10. Six hundred million three hundred thousand one hundred.

Separate into groups, read aloud, and then write in words:

1. 386	**2.** 38600	**3.** 3086	**4.** 386000
5. 308600	**6.** 3860	**7.** 30806	**8.** 308060
9. 1309	**10.** 309	**11.** 2007	**12.** 10008
13. 4007025	**14.** 139300	**15.** 247080	**16.** 7647200
17. 340600	**18.** 40706	**19.** 3008075	**20.** 400800

ROMAN NOTATION AND NUMERATION

Letters	I	V	X	L	C	D	M	Roman
Values	1	5	10	50	100	500	1000	Arabic

By combining these letters according to these rules, any number may be written:

I. When a letter is followed by the same letter or by one of less value, add the values of the letters.

Thus, II = 2; XX = 20; XV = 15; LX = 60.

II. When a letter is followed by one of greater value, subtract the letter of less value from the letter of greater value.

Thus, IV = 4; IX = 9; XL = 40; CD = 400.

1. Write 1776 in Roman notation.

1776 = 1000 + 700 + 70 + 6

M + DCC + LXX + VI = MDCCLXXVI

Write in letters:

2. 14	3. 25	4. 33	5. 54
6. 67	7. 89	8. 98	9. 5
10. 50	11. 55	12. 500	13. 505
14. 550	15. 555	16. 1620	17. 1680
18. 1700	19. 1840	20. 1900	21. 1911

22. Write MDCLXVI in Arabic notation.

$$\text{MDCLXVI} = \text{M} + \text{DC} + \text{LX} + \text{VI}$$
$$1000 + 600 + 60 + 6 = 1666$$

Write in figures:

23. XIX	24. XXVIII	25. XLIV
26. LXX	27. XCIX	28. MD
29. MM	30. MCM	31. MDCC
32. MCDLX	33. MDCXX	34. MDLXXIX
35. MLXVI	36. MCMX	37. MCMXVII

A dash over a letter increases its value one thousand fold. Thus, $\overline{\text{X}} = 10{,}000$; $\overline{\text{L}} = 50{,}000$.

38. Write these letters so that they shall represent 1000 times their ordinary value: I V C D M

39. Where are Roman numerals used?

Numbers may be expressed in three ways:
(1) by words; (2) by figures; (3) by letters.

40. In each of these three ways express your age; the number of days in a week; the number of months in a year; the number of weeks in a year; the number of minutes in an hour; the number of hours in a day; this page of the book; this year; the year in which you were born.

ADDITION *Oral*

6	2	7	9	3	1	0	4	8	5

Addition is the process of uniting two or more numbers into one number.

What is the sign of addition? What is it called? What does it show? What name is given to the result in addition?

Add 2 to each number in the diagram. Add 1; 3; 4; 5; 6; 7; 8; 9.

Find the sum of the numbers in the diagram. Beginning with 1; 2; 3; 4; 5; 6; 7; 8; 9, add the numbers in the diagram.

DRILL TABLE

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
1.	46	83	86	24	29	72	80	35	27	98
2.	85	77	48	71	33	96	14	79	12	20
3.	19	22	70	65	67	28	51	13	36	94
4.	63	66	64	59	42	50	25	87	58	11
5.	97	18	31	93	56	34	39	52	90	75
6.	92	30	55	47	78	61	73	26	74	49
7.	76	54	99	32	10	15	57	68	81	53
8.	38	41	23	16	84	89	62	60	45	37
9.	40	95	17	88	91	43	21	44	69	82

To each number in the table add 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12.

To each number add 10; 20; 30; 40; 50; 60; 70; 80; 90.

Give the sum of each number in the table and 87. Thus, $87 + 46$; $87 + 40 + 6$; think 127, 133; say 133.

Add other numbers to the numbers in the table.

Find the sum of column *A*. Of the other columns.

Find the sum of row 1. Of the other rows.

ADDING TWO COLUMNS AT ONCE *Sight*

Find the sum, adding both columns at once:
Thus in number 1: 11, 22, 31, 35, 45, 52, 64.
Test by adding down.

1.	2.	3.	4.	5.	6.
12	15	10	14	13	20
7	4	6	3	6	10
10	8	12	7	10	15
4	12	4	8	15	6
9	10	11	10	7	7
11	4	8	12	9	2
6	5	3	11	10	30
5	7	12	8	12	5

7.	8.	9.	10.	11.	12.
20	8	50	40	30	25
8	4	5	6	8	10
7	12	7	4	2	7
15	10	20	7	5	3
11	16	6	13	12	16
10	5	4	10	15	4
9	4	9	12	10	8
5	20	15	8	7	12
12	30	8	5	3	15
10	6	20	7	20	5

Beginning with 1; 2; 3: 4; 5; 6; 7; 8; 9; 10; 11; 12; 20; 21, add the numbers in examples 1 to 12.

Why are units written under units, tens under tens, etc., in adding numbers?

NOTE. Pupils should be required to prove work in fundamental processes until accuracy becomes a habit.

WRITTEN EXERCISES

	A	*B*	*C*	*D*	*E*
1.	285 +	374 +	562 +	134 +	658
2.	497 +	563 +	697 +	878 +	469
3.	174 +	481 +	148 +	306 +	542
4.	908 +	824 +	437 +	378 +	496
5.	734 +	512 +	657 +	498 +	763

Find the sum of each row; of all the rows; of each column; of all the columns.

	A	*B*	*C*	*D*	*E*	*F*
6.	$ 7 +	$800 +	$ 70 +	$749 +	$ 96 =	$——
7.	$ 89 +	9 +	900 +	60 +	875 =	——
8.	$856 +	76 +	6 +	800 +	50 =	——
9.	$ 80 +	964 +	48 +	8 +	600 =	——
10.	$500 +	90 +	788 +	57 +	5 =	——
11.	$ +	$ +	$ +	$ +	$ =	$——

Add and test:

1.	2.	3.	4.	5.
29	784	8437	48,950	576,432
92	392	6985	76,899	376,498
87	578	3800	73,204	507,837
96	767	3749	83,562	219,857
69	582	4527	87,420	436,849
56	479	5873	53,978	476,375
80	713	6734	37,739	278,135
82	427	8472	53,028	607,364

6.	7.	8.	9.	10.
$0.64	$ 60	$ 4.09	$ 90.09	$ 618.19
.08	9	.26	647.38	4.00
.05	409	37.68	.67	2753.80
.47	76	5.34	4.87	23.47
.29	8	.08	85.07	762.89
.35	847	.76	.29	4.68
.87	2256	56.34	918.60	2233.92

PROBLEMS *Written*

1. Find the total area of these possessions of the United States: Alaska, 590,844 square miles; Guam, 210 square miles; Hawaii, 6499 square miles; Panama Canal Zone, 474 square miles; Philippine Islands, 115,026 square miles; Porto Rico, 3435 square miles; and the Tutuila Group, 77 square miles.

2. In one month Mr. Blanchard paid $17.50 for rent, $14.87 for groceries, $12.68 for provisions, $2.94 for milk, $1.50 for ice, $3.75 for shoes, $7.55 for clothing, and $3.29 for other expenses. What was the total?

TABLE OF ELECTRIC RAILWAYS IN THE UNITED STATES IN 1908

	No. of Companies	Miles of Track	No. of Cars
New England states	140	5,390	15,210
Eastern states	460	12,063	35,951
Central states	354	14,723	24,350
Southern states	93	1,975	3,773
Western states	205	6,096	9,932

3. Find the total number of companies; miles of track used; cars operated.

4. There are in the United States 148 streams with a navigable length of 5305 miles flowing into the Atlantic, 54 with a navigable length of 7641 miles flowing into the Gulf, 53 with a navigable length of 11,483 miles flowing into the Mississippi, 2 with a navigable length of 315 miles flowing into Canadian waters, and 38 with a navigable length of 1606 miles flowing into the Pacific. Find the total number of navigable streams. The total number of miles navigable.

SUBTRACTION *Oral*

10 Subtraction is the process of taking one number from another, or of finding the difference between two numbers.

13

16 What is the sign of subtraction? What is it called? What does it show?

19 What is the minuend?

What is the subtrahend?

14 What is the result of subtraction?

17 How is work in subtraction tested?

Subtract 2 from each number in the diagram.

12 Subtract 3; 4; 5; 6; 7; 8; 9.

18 From the numbers in the drill table on page 5, take 2; 4; 6; 8; 1; 3; 5; 7; 9.

From 100 take each number in the drill table.

15 Give the difference between 48 and the numbers in the drill table. Between other numbers and the numbers in the drill table.

11

ADDITION AND SUBTRACTION *Sight*

Give the sum and the difference of:

1.	60	40	70	30	80	50	90	60	80	50
	32	17	45	18	31	23	54	46	39	15
2.	44	57	36	82	75	62	46	57	54	72
	35	28	19	38	36	29	18	38	29	33
3.	81	84	45	53	65	85	75	47	63	92
	27	45	18	27	46	57	28	29	26	57
4.	82	61	74	76	85	53	64	75	43	93
	53	37	39	48	28	19	28	47	18	39

Solve:

5.	$76 - 22 + 17$	6.	$26 + 73 - 59$
7.	$84 + 22 - 19$	8.	$67 - 29 + 68$
9.	$27 + 55 - 36$	10.	$48 + 36 - 39$
11.	$129 - 64 + 28$	12.	$73 - 25 + 46$
13.	$92 - 36 + 57$	14.	$55 + 38 - 46$
15.	$90 - 38 + 59$	16.	$120 + 82 - 36$
17.	$79 + 29 - 69$	18.	$153 - 69 + 118$
19.	$60 - 27 + 48$	20.	$115 - 49 + 57$
21.	$76 - 37 + 67$	22.	$139 + 65 - 49$
23.	$100 - 22 + 78$	24.	$111 - 77 + 88$

WRITTEN EXERCISES

Find differences, and test the work:

	A	*B*	*C*	*D*	*E*	*F*	*G*
1.	753	384	276	392	432	548	653
	347	159	98	37	184	193	579
2.	700	900	600	800	400	200	300
	351	243	108	356	297	73	132
3.	1762	3475	1432	3649	3987	5427	2731
	981	1956	666	1785	2898	3069	956
4.	9000	8000	3000	1000	7000	2000	6000
	347	652	283	187	1284	953	2243

5. How is the minuend found when the subtrahend and the remainder are given?

6. How is the subtrahend found when the minuend and the remainder are given?

Find the missing term :

	Minuend	Subtrahend	Remainder		Minuend	Subtrahend	Remainder
7.	637	150	?	8.	?	$1.96	$3.71
9.	?	325	187	10.	$6.42	$2.87	?
11.	476	?	232	12.	$5.50	?	$2.73

13. From 9000 take 9; 90; 900; 99; 909; 990; 999.

Solve :

14. $12.87 − $9.16
15. $129.37 − $56.84
16. $27.63 − $18.87
17. $58.74 − $29.38
18. $14.38 − $6.85
19. $112.49 − $60.82
20. $19.75 − $9.88
21. $378.50 − $195.84
22. $10.08 − $3.20
23. $632.70 − $360.95

PROBLEMS *Written*

1. The height reached by an aviator is 7912 feet. This is how many feet more than a mile (5280 feet)?

2. Mt. Everest is 29,002 feet high. This is how much more than five miles?

The area of the Great Lakes in square miles is: Superior, 31,200; Huron, 23,800; Michigan, 22,450; Erie, 9960; Ontario, 7240.

3. What is the difference in area between Lake Superior and each of the other lakes?

4. How many more square miles in Lake Huron than in Lake Ontario?

5. How many square miles does Lake Michigan lack of being equal in area to Lake Huron?

6. The area of Lake Erie and Lake Ontario together is how many square miles less than the area of Lake Michigan?

7. What is the total area of the Great Lakes?

8. The area of Illinois, 56,665 square miles, is how much greater than the area of New York, 49,204 square miles?

9. How many more than 100,000 square miles is the area of both Illinois and New York?

10. A city has $108,250; how much must it borrow to build a city hall to cost $300,000?

11. The receipts and expenses of five pieces of property for one year were as follows: No. 1, receipts $300, expenses $73.50; No. 2, receipts $450, expenses $126.85; No. 3, receipts $275, expenses $49.47; No. 4, receipts $360, expenses $80.76; No. 5, receipts $540, expenses $137.80. Find (1) the total receipts; (2) the total expenses; (3) the excess of the receipts over the expenses; this is the net income.

12. The total area of Maine, 33,040 square miles, New Hampshire, 9341 square miles, Vermont, 9564 square miles, Massachusetts, 8266 square miles, Rhode Island, 1248 square miles, and Connecticut, 4965 square miles, is how much less than the area of California, 158,297 square miles?

MULTIPLICATION *Oral*

Multiplication is the process of combining several equal numbers into one number.

What is the sign of multiplication? How is it read?

0
4
8
2
6
5
1
9
7
3

18 is the ——. What does it show?

4 is the ——. What does it show?

72 is the ——. What does it show?

What is the product of:

(1) 7 times 6? (2) 7 times 6 men? (3) 7 times $6?

The product always takes its name from the multiplicand.

What does the multiplier always show?

What kind of a number must the multiplier always be?

Multiply each of the numbers in the diagram by 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12.

Multiply by 20; 30; 40; 50; 60; 70; 80; 90.

Multiply by 10:

10 60 20 70 50 30 80 40 90

Multiply by 20; 40; 60; 80; 30; 50; 70; 90.

Multiply the numbers in the drill table on page 5 by 2; 3; 4; 5; 6; 7; 8; 9. Multiply in this way: 8 times 74 = 8 times 70, + 8 times 4. Think 560, 32, 592.

WRITTEN EXERCISES

1. Multiply 56 by 18.

56
18
448
56
1008

Work in this multiplication may be tested in three ways:

(1) By multiplying 18 by 56. 56 times 13 = 1008.

Since 1008 is the product of the two factors 18 and 56, 1008 divided by either factor will give the other factor.

(2) By dividing 1008 by 18. $1008 \div 18 = 56$.

(3) By dividing 1008 by 56. $1008 \div 56 = 18$.

Find products, and test:

	A	*B*	*C*	*D*	*E*	*F*	*G*
2.	29	67	39	68	78	88	47
	58	84	17	42	97	66	35
3.	347	428	562	497	863	892	439
	247	432	368	263	892	157	284
4.	$0.47	$0.96	$0.18	$0.35	$0.27	$0.87	$0.07
	94	137	208	118	267	654	905
5.	$82.94	$56.78	$90.72	$34.56	$78.90	$76.08	$39.75
	408	549	675	246	468	795	658

$3 \times 4 \times 5 = ?$ $4 \times 5 \times 3 = ?$ $5 \times 3 \times 4 = ?$ $5 \times 4 \times 3 = ?$

Does it make any difference in what order numbers are multiplied together?

Find the product, selecting multipliers to make your work as easy as possible:

6. $50 \times 27 \times 4$	**7.** $45 \times 9 \times 2$	**8.** 1375×806
9. $28 \times 25 \times 2$	**10.** $4 \times 87 \times 15$	**11.** 6307×534
12. $57 \times 15 \times 4$	**13.** $65 \times 20 \times 5$	**14.** 4386×402
15. $96 \times 8 \times 5$	**16.** $150 \times 37 \times 2$	**17.** 408×8275
18. $72 \times 75 \times 2$	**19.** $28 \times 25 \times 4$	**20.** 3258×407

21. Find the product of 758 and 65,942.

22. Multiply 78,659 by 4037.

23. Make a table of the results and memorize:

$11 \times 11 =$ —	$16 \times 16 =$ —	$21 \times 21 =$ —
$12 \times 12 =$ —	$17 \times 17 =$ —	$22 \times 22 =$ —
$13 \times 13 =$ —	$18 \times 18 =$ —	$23 \times 23 =$ —
$14 \times 14 =$ —	$19 \times 19 =$ —	$24 \times 24 =$ —
$15 \times 15 =$ —	$20 \times 20 =$ —	$25 \times 25 =$ —

Name the two equal factors of:

24.	4	9	16	25	36	49	64	81
25.	100	400	121	361	441	144	324	484
26.	225	625	196	256	576	169	289	529

NOTE. Drill should be given on exercises 23, 24, 25, and 26, until pupils can give results instantly.

PROBLEMS *Written*

1. How much will 48 tons of hay cost at $23 a ton?

2. How many gallons of oil in 48 barrels of 42 gallons each?

3. How many yards of cloth in 35 cases if each case contains 72 pieces of 28 yards each?

4. How much must be paid for 68 sheep at $5 each, and 2 cows at $65 each?

5. Mr. Chase leases a house for 4 years at $45 a month. What is the rent for the entire time?

6. It took 27 days for 14 men, working 8 hours a day, to lay a water pipe. This was how many hours' labor?

7. In gathering his crops, Mr. Fitch hires 4 men at $1.85 a day and 2 boys at $0.75 a day. How much does he pay for labor in 4 weeks?

8. An acre of land contains 43,560 square feet. How much is it worth at 7 cents a square foot?

9. Rocky Ford melons are shipped from Colorado in crates of 45 melons each, 300 crates to a car. How many melons in 8 carloads?

CASH ACCOUNTS

Alfred kept a cash account; that is, an account of money received and paid out. At the top of the left-hand page of his cash book he wrote Receipts. On this page he put down how much he had on hand and how much he received. At the top of the right-hand page he wrote Expenses. On this page he put down how much he paid out.

This was his account for one week:

1911		Receipts			1911		Expenses		
June	1	On hand	$1	28	June	2	Ball game		25
	3	Cutting lawn		60			Blank book		10
		Aunt Sarah		50		3	Pencils		08
		Allowance		25		5	Car fares		15
						6	Rabbit		25
							Fountain pen	$1	15
							Balance		65
			$2	63				$2	63

1. How much money had he during the week?

2. How much did he spend?

3. What was the difference between his receipts and his expenses?

4. Write his account for the next week, in which he cut two lawns, one for 50 cents, the other for 60 cents; sold his rabbit for 40 cents; received 20 cents for running errands, and his allowance of 25 cents; spent 50 cents at the circus, 15 cents for peanuts, 75 cents for a trapeze, and 67 cents for a dog collar. Use any dates between

June 8 and 14 that you wish. What item must be put down first on the left-hand page?

5. Another boy had 43 cents on hand at the beginning of a week. He earned $1.85 selling papers, 40 cents running errands for the grocer, and 75 cents taking care of a furnace. He spent 65 cents for an arithmetic, 5 cents for paper, 3 cents for a pencil, and 25 cents for a cap. Write his cash account.

6. Keep your cash account for one week and show it to your parents.

NOTE. Imaginary cash accounts should be made from data furnished by pupils.

PARTS OF 100 *Oral*

1. What part of 100 is 50? 5? 25? 75? 20? 40? 60? 80? 10? 30? 70? 90?

2. Learn:

$33\frac{1}{3}$ is $\frac{1}{3}$ of 100	$66\frac{2}{3}$ is $\frac{2}{3}$ of 100
$16\frac{2}{3}$ is $\frac{1}{6}$ of 100	$83\frac{1}{3}$ is $\frac{5}{6}$ of 100
$12\frac{1}{2}$ is $\frac{1}{8}$ of 100	$37\frac{1}{2}$ is $\frac{3}{8}$ of 100
$62\frac{1}{2}$ is $\frac{5}{8}$ of 100	$87\frac{1}{2}$ is $\frac{7}{8}$ of 100

3. What part of a dollar is 90¢? $87\frac{1}{2}$¢? $83\frac{1}{3}$¢? 80¢? 75¢? 70¢? $66\frac{2}{3}$¢? $62\frac{1}{2}$¢? 60¢? 50¢? 40¢? $37\frac{1}{2}$¢? $33\frac{1}{3}$¢? 30¢? 25¢? 20¢? $16\frac{2}{3}$¢? $12\frac{1}{2}$¢? 10¢? 5¢? 2¢? 1¢?

SHORT METHODS IN MULTIPLICATION *Oral*

1. What is the cost of 8 neckties at 50 ¢ each?

SOLUTION. At $1 each, they would cost $8. At 50¢ each, they will cost $\frac{1}{2}$ of $8, or $4.

Find the cost of:

2. 12 caps @ 25¢
3. 28 yards of cloth @ 75¢
4. 40 plates @ $37\frac{1}{2}$¢
5. 72 bushels of grain @ $87\frac{1}{2}$¢
6. 36 brushes @ $33\frac{1}{3}$¢
7. 48 pairs of slippers @ $62\frac{1}{2}$¢
8. 32 books @ $62\frac{1}{2}$¢
9. 18 bushels of apples @ $66\frac{2}{3}$¢
10. 30 pans @ $16\frac{2}{3}$¢
11. 17 pairs of gloves @ 50¢

12. At 75 cents a bag, what will 16 bags of flour cost?

13. Mrs. Dodge bought 16 yards of dress goods at $62\frac{1}{2}$¢ a yard. How much did she pay?

14. How much must be paid for 100 skeins of yarn at $12\frac{1}{2}$¢ each?

15. At $87\frac{1}{2}$¢ each, how much will 2 dozen pieces of sheet music cost?

Make and solve problems based on the following:

	Number of Articles	Price of Each		Number of Articles	Price of Each
16.	24	50¢	17.	60	$0.25
18.	32	75¢	19.	64	$0.37\frac{1}{2}$
20.	40	$87\frac{1}{2}$¢	21.	48	$0.66\frac{2}{3}$
22.	54	$16\frac{2}{3}$¢	23.	40	$0.62\frac{1}{2}$
24.	27	$33\frac{1}{3}$¢	25.	56	$0.12\frac{1}{2}$
26.	15	50¢	27.	14	$0.25
28.	18	$83\frac{1}{3}$¢	29.	12	$0.12\frac{1}{2}$

30. Multiply 36 by 50.

100 times 36 = 3600. 50 is $\frac{1}{2}$ of 100. Then 50 times 36 is $\frac{1}{2}$ of 100 times 36, or 1800. Think 36, 3600, 1800.

Multiply by 50; 25:

31. 48 32 64 52 40 80 60 28 72 84

32. 24 20 36 76 68 56 44 88 90 60

Multiply by $12\frac{1}{2}$; 25; 50:

33. 8 16 32 40 56 64 80 48 72 96

Multiply by $33\frac{1}{3}$; $16\frac{2}{3}$; 50:

34. 6 12 24 18 48 30 72 36 42 60

DIVISION *Oral*

Division is the process of finding how many times one number is contained in another number, or of finding one of the equal parts of a number.

What is the sign of division? What does it show?

What is the dividend? Divisor? Quotient?

When 1 hat costs \$3, how is the number that can be bought for \$24 found?

When 8 hats cost \$24, how is the cost of 1 found?

How is work in division tested:

(1) When there is no remainder? Illustrate with 54 divided by 9.

(2) When there is a remainder? Illustrate with 54 divided by 5.

Divide the numbers in the drill table on page 5 by 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12.

DRILL IN DIVISION *Written*

Divide 1645 by 47 and test your work.

$$47\overline{)1645}\quad 35$$

This division may be tested in two ways:

(1) By finding the product of 47 and 35. The result equals the dividend.

(2) By dividing 1645 by 35. The result is the divisor.

What is the dividend when the quotient is 37 and the divisor 79?

I. Divide (short division) each dividend in the examples below by 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12.

II. Solve:

	A	*B*	*C*
1.	735 ÷ 32	502 ÷ 28	873 ÷ 123
2.	936 ÷ 27	753 ÷ 45	927 ÷ 204
3.	438 ÷ 19	820 ÷ 33	624 ÷ 157
4.	675 ÷ 28	908 ÷ 26	738 ÷ 236
5.	876 ÷ 37	627 ÷ 35	875 ÷ 352
6.	3647 ÷ 56	3760 ÷ 59	9801 ÷ 262
7.	3579 ÷ 42	5642 ÷ 97	5624 ÷ 371
8.	8246 ÷ 73	2746 ÷ 78	9000 ÷ 268
9.	1357 ÷ 45	8400 ÷ 36	5628 ÷ 537
10.	9246 ÷ 56	5207 ÷ 49	4835 ÷ 942
11.	8157 ÷ 97	7000 ÷ 73	2764 ÷ 127
12.	4321 ÷ 39	7465 ÷ 58	6408 ÷ 809
13.	8901 ÷ 64	8400 ÷ 86	3769 ÷ 543
14.	5730 ÷ 82	3498 ÷ 39	6437 ÷ 276
15.	5087 ÷ 75	4356 ÷ 45	7500 ÷ 324
16.	84,327 ÷ 64	72,468 ÷ 352	187,604 ÷ 427
17.	73,820 ÷ 87	93,807 ÷ 654	537,416 ÷ 609
18.	17,600 ÷ 96	76,491 ÷ 326	806,320 ÷ 347
19.	30,874 ÷ 52	47,000 ÷ 742	943,076 ÷ 465
20.	56,792 ÷ 48	86,308 ÷ 564	706,321 ÷ 857

21. How many times is 309 contained in 13,596?

22. By what must 2576 be divided to get a quotient of 46?

23. In dividing a number by 9, the quotient is 100 and the remainder 7. What is the number?

24. In dividing 1247 by a certain number, the quotient is 37 and the remainder 26. What is the divisor?

25. What is the dividend when divisor and quotient are both 48?

26. What is one of the 27 equal parts of 1161?

27. What is $\frac{1}{58}$ of 4872?

28. Six hundred eighty-four is made up of three factors. Two of the factors are 9 and 19. What is the third?

SHORT METHODS IN DIVISION *Oral*

1. How many times is 50 cents contained in 1 dollar?

2. 25 cents is contained how many times in 1 dollar? In 50 cents? In 75 cents?

3. $12\frac{1}{2}$ cents is contained how many times in 1 dollar? In 50 cents? In 25 cents? In 75 cents?

4. $33\frac{1}{3}$ cents is contained how many times in 1 dollar?

5. How many times is $16\frac{2}{3}$ cents contained in 1 dollar? In 50 cents?

6. Divide \$2 by $12\frac{1}{2}$ cents.

$12\frac{1}{2}$ cents is contained —— times in \$1; in \$2 it is contained —— times.

Using a short method, divide:

7. \$8 by 25¢ **8.** \$1.50 by 25¢ **9.** \$2.75 by 25¢

10. \$6 by 50¢ **11.** \$2.50 by 50¢ **12.** \$3.50 by 50¢

13. \$4 by $16\frac{2}{3}$¢ **14.** \$2.50 by $16\frac{2}{3}$¢ **15.** \$4.50 by $16\frac{2}{3}$¢

16. \$5 by $33\frac{1}{3}$¢ **17.** \$7.00 by $33\frac{1}{3}$¢ **18.** \$10.00 by $33\frac{1}{3}$¢

19. \$3 by $12\frac{1}{2}$¢ **20.** \$0.75 by $12\frac{1}{2}$¢ **21.** \$1.25 by $12\frac{1}{2}$¢

PROBLEMS *Oral*

1. I have $12. How many knives can I buy at 50¢ each? $33\frac{1}{3}$¢? $12\frac{1}{2}$¢? 25¢? $16\frac{2}{3}$¢?

2. How many handkerchiefs at 25 cents each can be bought for $2? $3.50? $0.75? $1\frac{1}{4}$? $6?

3. How many yards of crash at $12\frac{1}{2}$ cents a yard does a bill of $3 represent? A bill of $1.25? $5? $3.75? $2.50?

4. When eggs are 25¢ a dozen, a case costs $7.50. How many dozen in a case?

5. A shipment of paper costs $12. The rate is $12\frac{1}{2}$ cents a pound. What is the weight?

6. At $16\frac{2}{3}$ cents an hour, a boy working on a farm earned $7. How many hours did he work?

7. At $33\frac{1}{3}$ cents an hour, his father earned $15. How many hours did he work?

DIVISORS AND DIVIDENDS ENDING IN 0 *Written*

240 ÷ 30	2400 ÷ 300	98,000 ÷ 2000
3Ø)24Ø	3ØØ)24ØØ	2ØØØ)98ØØØ
8	8	49

Solve:

1. 980 ÷ 70	2. 1360 ÷ 80	3. 5100 ÷ 300
4. 1760 ÷ 80	5. 1170 ÷ 90	6. 7600 ÷ 400
7. 700 ÷ 50	8. 1020 ÷ 60	9. 8400 ÷ 700
10. 1280 ÷ 80	11. 850 ÷ 50	12. 7800 ÷ 600
13. 1440 ÷ 60	14. 2700 ÷ 90	15. 108,900 ÷ 900

16. How many tons of hay in a car of 38,000 pounds?

Find the number of tons of fertilizer in:

17. 18,000 lb.	**18.** 29,300 lb.	**19.** 225,000 lb.
20. 48,000 lb.	**21.** 37,400 lb.	**22.** 637,000 lb.
23. 75,000 lb.	**24.** 19,500 lb.	**25.** 861,000 lb.

PROBLEMS *Written*

1. At \$15 a ton, a bill for hay was \$405. How many tons were bought?

2. Mr. Flagg sold 19 acres of land for \$1482. How much did he receive an acre?

3. Twenty-eight bushels of oats were bought for \$15.68. What was the rate per bushel?

4. At \$0.35 each, how many books can be bought for \$9.45?

$$.35\overline{)9.45}$$

$$35\overline{)945}\quad 27$$

When the divisor contains cents, change both dividend and divisor to cents.

5. A fruit dealer paid \$77 for oranges at \$2.75 a box. How many boxes did he buy?

6. A florist purchased lily bulbs at \$0.25 each. His bill was \$12.75. How many did he buy?

7. How many sleds at \$1.35 each does a bill of \$64.80 represent?

8. A sale of shovels at 85 cents each yielded \$44.20. How many were sold?

9. A hardware dealer's bill for saws was \$104.40. If each saw cost 87 cents, how many were bought?

10. How many peach trees at \$2.25 each can be bought for \$29.25?

11. A man asked \$1.75 a day for his work on a farm. He received \$47.25. How many days did he work?

12. At \$7.25 per cord, how many cords of wood can be bought for \$195.75?

RATIO

Oral

1. What is the relation of 24 to 6? $24 \div 6$, or 4.

2. What is the relation of 6 to 24? $6 \div 24$, or $\frac{1}{4}$.

The relation of one number to another is their ratio.

Ratio is indicated by the sign : placed between the two numbers or quantities, and is equivalent to the sign $\div$.

Find the ratios:

3. 10 : 2	**4.** 15 : 3	**5.** 20 : 5	**6.** 32 : 8
7. 4 : 12	**8.** 7 : 42	**9.** 9 : 63	**10.** 8 : 48
11. 2 : 3	**12.** 8 : 4	**13.** 5 : 8	**14.** 6 : 9
15. 12 : 20	**16.** 10 : 15	**17.** 15 : 6	**18.** 20 : 15

19. 9 books : 63 books

20. 56 days : 7 days

21. 8 dollars : 72 dollars

22. 40 bushels : 5 bushels

23. 27 days : 30 days

24. 24 pencils : 60 pencils

25. 32 trees : 80 trees

26. 27 cents : 45 cents

27. 5 days to a week. (Since only things of the same kind can be compared, change the week to days.)

28. 3 days : 2 weeks

29. 3 quarts : 1 gallon

30. 2 bushels : 1 peck

31. 27 inches : 1 yard

32. 2 years : 1 month

33. 6 feet : 8 inches

34. $1\frac{1}{2}$ feet : 1 yard

35. 30 minutes : $2\frac{1}{2}$ hours

36. James buys corncakes at the rate of 3 for 5 cents. How much does he pay for a box of 60?

37. Mrs. Arnold bought 12 quarts of berries for *75 cents.* What would 4 quarts cost at the same rate?

f a farmer raises 7 tons of hay on 5 acres of land, ıght he to raise on 35 acres?

f 18 bushes produce $12 worth of roses, how many will produce $72 worth?

RECEIPTS

'0 *Chicago, Mar. 1, 1910.*

eceived from *Arthur Y. Waite*

Twenty x/100 *Dollars*

ıt of house at No. 24 Mason St.

Mar. 1, 1910, to April 1, 1910.

Charles Z. Young.

paid money? How much? For what? To When? What shows that money has been paid? paper is a *receipt.*

r the receipt until you can write a similar one.

should receipts always be preserved?

_____ 19____

:eived from_____

_____ /100 *Dollars*

1. This shows a blank receipt such as can be bought at any stationery store. Copy the form and fill in the blanks as you would in giving a receipt to one of your schoolmates who hires your cart for 50 cents.

2. Write a receipt for $150 received for the sale of a gray horse.

3. Suppose that you own and rent a house for $25 a month. Write a receipt for a month's rent.

4. You sold your bicycle for $12.50. Write the receipt.

5. Write a receipt for 3 months' work on a farm at $20 per month.

BILLS

St. Louis, July 17, 1910.

Mr. James K. Bryant

Bought of STEVENS & WRIGHT

July 1	2½ lb. Steak	@	$0.28
" 3	1¼ lb. Ham	@	.25
" "	4 Cucumbers	@	.05
" 6	2 boxes Berries	@	.17
" "	½ peck Peas	@	.50

Received payment,
Stevens & Wright.
By Harper.

When were the purchases made? By whom? From whom? Cost of each item? Amount of the bill?

What shows that the bill has been paid?

Was payment made directly to the owners of the store or to one of their clerks?

When a bill contains the words "Received payment" followed by the signature of the seller or his agent, it is a *receipt.*

There are two parties in a sale — the debtor and the creditor.

The debtor is the one who buys; he owes something.

The creditor is the one who sells; he is one to whom something is owed.

Who is the debtor in the above bill? Creditor?

1. Complete this bill:

CLEVELAND, Jan. 16, 1911.

Mr. GARFIELD S. PARKER

Bought of THE STRONG MANUFACTURING CO.

200 Hammers	@	$0.18			
50 3″ Chisels	@	.42			
50 2″ Chisels	@	.32			
50 1½″ Chisels	@	.28			

Notice that when the articles are all bought at one time, the date of the bill is the date of purchase.

2. The equipment for the sewing class was: 4 dozen pairs of scissors at 20 cents each, 4 dozen measures at 2 cents apiece, 6 5-cent papers of needles, 6 papers of pins at 8 cents each, 100 spools of thread at 3 cents each, and 2 dollars' worth of cloth. Make out the bill. Buyer, the city in which you live. Seller, yourself.

3. Alice made a dress, using these materials: 1 pattern @ 15 cents, 8 yards of gingham @ 15 cents, 1 spool of

thread @ 5 cents, 1½ dozen buttons @ 10 cents, and 5 yards of lace @ 12 cents. Make out the bill.

4. The boys of a baseball nine bought 4 bats @ $0.50, 2 balls @ $0.75, a catcher's mask for $2.75, and 4 mitts @ $0.80. Make out the bill to the treasurer of the club. They shared the cost equally. How much did each pay?

5. Imagine that you are going on a journey and buy collars, shoes, and umbrella. Make out the bill.

6. Make out the bill for three things you buy for your mother from the grocer.

7. Make out Mr. C. K. Austin's milk bill for October, supposing that he takes 1 quart a day at 8 cents a quart, and during the month has 1½ quarts of cream at 30 cents a quart.

8. Ten boys and girls are going on a picnic, and one of them is selected to buy things they need. Make out the bill for four things you think he will buy.

9. Make out other bills for goods purchased at different stores, using the prices given in the daily paper.

DICTATION EXERCISES

1. $12 \times 9, +2, \div 11, \times 4, -8, \div 8, \times 9, +9, +5, \div 10.$
2. $54 \div 6, +3, \times 5, +2, -2, \div 7, +12, \times 3, -6, \div 7.$
3. $72 \div 6, -3, \times 6, +2, \div 7, +12, -5, \div 3, +8, \times 2.$
4. $16 \times 4, +2, \div 3, +8, \div 5, \times 7, -6, \div 9, +11, \times 3.$
5. $80 - 5, \div 15, +9, \times 2, \div 4, +8, \div 5, \times 7, +6, \div 9.$
6. $144 \div 12, +6, \times 5, \div 9, +11, \div 3, +5, \times 9, -12, \div 8.$
7. $84 \div 7, +8, \times 4, -5, \div 25, \times 8, -3, \div 7, +16.$
8. $56 \div 8, \times 4, \div 2, -6, \times 9, +3, \div 5, -8, \times 9, -7.$
9. $7 \times 7, +5, \div 6, \times 3, +8, \div 5, +2, \times 8, -6, \div 11.$

10. $9 \times 9, -6, +5, \div 20, -1, \times 8, +9, \times 2, \div 11, \times 5.$

11. $8 + 7, \div 3, -5, +6, \times 9, +2, \div 8, \times 6, +3, \div 9.$

12. $64 \div 8, +7, \times 4, \div 3, +8, \div 7, \times 9, -20, \times 3, \div 4, \times 0.$

13. $6 + 8, -10, \times 9, \div 3, +12, -6, +2, \div 5, -4, +17.$

14. $9 \times 5, \div 3, +6, \div 7, +9, \times 8, \div 3, +3, \div 7, \times 0.$

15. $6 \times 3, +7, \div 5, \times 3, +5, \div 4, +13, \div 6, +7, \times 12.$

Similar examples should be dictated daily.

MISCELLANEOUS PROBLEMS *Written*

1. Lake Ontario is 234 feet above sea level, Lake Erie is 330 feet higher than Lake Ontario, Lake Huron 10 feet higher than Lake Erie, Lake Michigan 4 feet higher than Lake Huron, and Lake Superior 22 feet higher than Lake Michigan. How many feet above sea level is Lake Superior?

A leaf from the time book of Mr. Blake, a contractor, showing the number of hours each employe worked each day for a week, and the pay of each per hour.

	M.	T.	W.	T.	F.	S.	Total Hours	Pay per Hour	Amount Due for Week's Work
C. E. Austin	8	8	7	4	6	8		$.35	$
A. P. Dudley	8	8	8	8	8	8		.30	
H. G. Preston	6	8	3	8	5	8		.40	
O. L. Jackson	8	8	4	6	5	7		.25	
T. M. Perry	3	4	8	5	7	8		.27	
								Total	$

2. Find (1) the total number of hours each man worked; (2) the wages of each for the week; and (3) total pay of all.

3. A plumber receives 80 cents an hour for a day's work of 8 hours, and double pay for overtime. How much does he earn in a week when he works 6 hours overtime?

4. What are the average daily earnings of a boy who receives \$0.88, \$0.25, \$1.15, \$0.75, \$0.50, and \$0.61 in one week?

5. One suit of clothes cost \$12.75 and another twice as much. What did both cost?

6. Two farm wagons cost \$50. One cost \$27.50. How much did the other cost?

7. What is the cost of 8 plows at \$6.75 each?

8. How many churns at \$6.75 each can be bought for \$108?

9. 18 watches cost \$135; what is the price of one?

10. What is the gain on a dozen cans of tomatoes bought at \$1.32 per dozen and sold at 15 cents apiece?

11. When 75 pounds of ham are bought for 24 cents a pound, for how much must the lot be sold to gain 4 cents a pound?

12. With money received by a will a man bought a house for \$3950 and 16 acres of land at \$125 an acre. What was the amount of the legacy?

13. Mr. Holt raised 288 bushels of rye on 24 acres of land. At the same yield per acre, how many bushels did he get the next year from a field of 8 acres?

14. A dealer in farm supplies paid \$675 for hay rakes at \$18.75 each. How many did he buy?

15. At another time he bought 28 potato diggers for \$567. How much apiece?

16. If he paid $18.75 apiece for cotton planters and sold them at $25 each, what was his gain on 36?

17. In one month he made $180 by buying disc cultivators at $22.50 and selling at $30. How many did he sell?

18. At $1.28 each, how much will 24 umbrellas cost?

19. What is the cost of 15 tons of coal at $6.75 a ton, and 6 cords of wood at $7.50 a cord?

SALES IN A DEPARTMENT STORE

	Dry Goods		Shoes		Gloves	
	Cash	Charge	Cash	Charge	Cash	Charge
Mon.	$297.15		$416.87		$120.49	
		$168.70		$197.00		$ 69.38
Tu.	455.80		115.00		189.15	
		237.63		378.18		112.27
Wed.	264.39		237.08		205.45	
		308.07		198.14		37.49
Th.	317.83		345.42		117.13	
		359.03		275.00		203.86
Fri.	453.75		100.90		269.00	
		576.24		109.73		158.08
Sat.	689.29		346.80		200.78	
		190.87		538.73		217.90

20. Find:

(1) Total cash sales in each department for the week.
(2) Total charge sales in each department for the week.
(3) Total cash sales on each day of the week.
(4) Total charge sales on each day of the week.
(5) Total sales in each department for the week.
(6) Total sales in all departments for the week.
(7) Total sales in all departments for each day.
(8) Total daily sales in all departments for the week.

21. A telephone rental is $3.50 a month. What is the yearly rental?

22. A business man pays yearly $36 for his office telephone, and $27 for his house telephone. How much does he pay every month?

23. Boxwood 2-foot rules are bought at 8 cents and sold for 10 cents. How many must be sold to gain one dollar?

24. At $1.62 a yard, a piece of silk cost $84.24. How many yards in the piece?

FACTORS *Oral*

1. Name five numbers between 10 and 100 and tell their factors.

2. What are the factors of a number?

A number that can be separated into factors is a composite number.

3. Name the composite numbers below 26; between 26 and 47; 47 and 73; 73 and 100.

A number that cannot be separated into factors is a prime number.

4. Name the prime numbers below 25; between 25 and 50; between 50 and 75; between 75 and 100.

A prime number used as a factor is a prime factor.

5. Name the prime factors of 60.

We may think of 60 as 6×10. The prime factors of 6 are 2 and 3; the prime factors of 10 are 2 and 5; therefore, the prime factors of 60 are 2, 2, 3, and 5.

6. Name the prime factors of:

28 36 40 48 54 56 72 80 81 144

NOTE. Going rapidly around the class, let the pupils recite as follows: 1 is a prime number; 2 is a prime number; 3 is a prime number;

4 is a composite number, its prime factors are 2 and 2; 5 is a prime number; 6 is a composite number, its prime factors are 2 and 3; and so on to 100.

When several equal factors occur in the answer, a small figure, called an *exponent*, is written at the right and a little above the factor to show how many times the factor is used. Thus, 2^3 means that 2 is used as a factor three times. $2 \times 2 \times 2 = 8$.

The factors of 72—2, 2, 2, 3, 3—are written $2^3 \times 3^2$.

7. What number does 5^2 represent? 2^5?

8. 2×5^2 are the prime factors of what number?

9. Of what number are $2^2 \times 3^2$ the prime factors?

10. The prime factors of a number are $2^2 \times 3 \times 5^2$. What is the number?

Written

11. What are the prime factors of 168?

2	168
2	84
2	42
3	21
	7

To find the prime factors of a number not readily factored by inspection, we divide the number by one of its prime factors; then divide the resulting quotient by one of its prime factors, and continue the division until the resulting quotient is prime. The several divisors and the last quotient are the prime factors.

Ans. $2^3 \times 3 \times 7$.

Find the prime factors of:

12. 125 **13.** 126 **14.** 135 **15.** 136 **16.** 165

17. 245 **18.** 252 **19.** 288 **20.** 296 **21.** 268

GREATEST COMMON DIVISOR

Oral

1. What is the greatest number that will exactly divide 12 and 18?

The greatest number that will exactly divide two or more numbers is their greatest common divisor (g.c.d.).

Name the greatest common divisor of:

2. 16, 24	3. 18, 27	4. 32, 40	5. 28, 42
6. 42, 56	7. 36, 54	8. 35, 56	9. 21, 32
10. 6, 8, 12	11. 9, 12, 15	12. 12, 15, 18	
13. 12, 20, 30	14. 6, 15, 27	15. 10, 25, 30	
16. 9, 27, 45	17. 12, 15, 24	18. 21, 28, 35	
19. 18, 24, 30	20. 32, 48, 60	21. 36, 54, 72	

Written

22. What is the greatest common divisor of 48, 84, and 90?

2	48	84	90
3	24	42	45
	8	14	15

$2 \times 3 = 6$, the g. c. d.

When the g. c. d. is not readily found by inspection, use this method. Arrange the numbers in a row and successively divide them by any number that will exactly divide all of them. Repeat the process with the resulting quotients. Continue the division until there is no number that will exactly divide all of the quotients. The product of the several divisors is the greatest common divisor. 2 and 3 are common divisors of all the numbers. Their product, 6, is the greatest common divisor of the numbers.

Find the greatest common divisor of:

23. 42, 63, 84	24. 27, 63, 81	25. 32, 64, 96
26. 42, 70, 98	27. 54, 72, 90	28. 32, 56, 84
29. 35, 42, 91	30. 84, 96, 144	31. 36, 54, 72

LEAST COMMON MULTIPLE

Oral

A multiple of a number is a number obtained by using that number as a factor. Thus, 2, 4, 6, 8, 10, etc., are multiples of 2.

1. Name some multiples of 3.
2. Name some multiples of both 2 and 3.

Since 6, 12, 18 are multiples of both 2 and 3, they are common multiples of 2 and 3.

A number that is a multiple of two or more numbers is a common multiple of the numbers.

The least multiple common to two or more numbers is their least common multiple (l. c. m.).

Name the least common multiple of:

3. 7 and 8 **4.** 4 and 10 **5.** 6 and 9 **6.** 8 and 6

7. 8 and 12 **8.** 9 and 12 **9.** 10 and 15 **10.** 12 and 15

Written

11. Find the least common multiple of 8, 10, and 12.

$$\begin{array}{r|lll} 2 & 8 & 10 & 12 \\ \hline 2 & 4 & 5 & 6 \\ \hline & 2 & 5 & 3 \end{array}$$

$2 \times 2 \times 2 \times 5 \times 3 = 120$

When the least common multiple is not readily seen, use this method.

Divide the numbers by any factor common to two or more of them. Continue the division of the resulting quotients until no two of them have a common factor. The divisors and the remaining quotients are the factors of the least common multiple.

Find, in the easiest way possible, the least common multiple of:

12. 12, 28 **13.** 18, 24 **14.** 16, 20 **15.** 20, 35

16. 8, 10, 12 **17.** 12, 16, 20 **18.** 15, 20, 40 **19.** 16, 24, 30

20. 9, 12, 15 **21.** 12, 15, 18 **22.** 16, 20, 24 **23.** 15, 18, 24

USE OF SIGNS

Operations in arithmetic are indicated by signs. The signs most commonly used are $+$, $-$, $\times$, $\div$.

When several operations are indicated in the same expression, operations indicated by $\times$ and $\div$ are performed

before operations indicated by + and −. Thus, in $8+6 \times 2$, first multiply 6 by 2, and then add their product to 8. $8+6\times 2 = 8+12 = 20$.

Solve:

1. $24-8\times 2$	2. $24\div 8+2$	3. $20-8\div 2$
4. $27\div 3+3$	5. $12\times 8+4$	6. $12+8\times 4$
7. $9+3\times 2$	8. $36\div 4+8$	9. $36-8\div 4$
10. $16+4\times 8$	11. $16\div 2+2$	12. $16-2\times 2$

13. $2+3+4\times 5$
14. $2\times 3+4\times 5$
15. $2\times 3\times 4+5$
16. $24\div 3+9\div 3$
17. $18-3+3\times 2$
18. $7\times 2+21\div 3$
19. $24+3\times 9-3$
20. $18-9\div 3\times 2$
21. $18\div 9+3-2$
22. $18\div 9+3\times 2$

23. Which is larger, $10\div 10+10$ or $10-10\div 10$?

24. Show that $42\div 6+7$ is equal to $42-7\times 4$.

25. What is the value of $4\times 5+6\times 7-8\div 4$?

26. What is the sum of $66\div 6+5$ and $66-5\times 10$?

27. What is the product of $8+56\div 8$ and $56-12\times 4$?

28. How many times is $32\div 8+5$ contained in $32+8\times 5$?

CANCELLATION

1. Divide $7\times 8\times 12$ by 4×6.

$$\frac{7\times \overset{2}{\cancel{8}}\times \overset{2}{\cancel{12}}}{\underset{1}{\cancel{4}}\times \underset{1}{\cancel{6}}} = 28$$

2. What factors are common to both dividend and divisor?

3. Does striking out or canceling equal factors from *both dividend* and divisor affect the quotient?

Divide:

4. $12 \times 10 \times 9$ by 4×5
5. $24 \times 15 \times 6$ by $8 \times 5 \times 3$
6. $14 \times 20 \times 6$ by $7 \times 10 \times 2$
7. $18 \times 60 \times 15$ by $9 \times 5 \times 20$
8. $15 \times 44 \times 18$ by $22 \times 5 \times 9$
9. $36 \times 32 \times 16$ by $24 \times 3 \times 8$
10. $64 \times 35 \times 42 \times 40$ by $8 \times 56 \times 12 \times 10$
11. $72 \times 81 \times 45 \times 28$ by $90 \times 144 \times 63$

12. $\dfrac{100 \times 75 \times 50 \times 17}{150 \times 25 \times 4}$ 13. $\dfrac{48 \times 36 \times 24}{1728}$ 14. $\dfrac{21120}{4752}$

DRILL EXERCISE IN RAPID ADDITION AND SUBTRACTION

10	8	7	13	5	4	21	11	7	9
5									12
12									7
8									10
6									8
15									31
10									7
7									10
6									6
9	4	12	7	20	15	8	6	11	8

Beginning with any number in the margin and going in either direction, rapidly add the numbers until 100 or any given number is reached.

Beginning with 100 or any given number, rapidly subtract the successive numbers.

FRACTIONS

A unit is a single thing; as 1 apple.

A fraction is one or more of the equal parts of a unit; as $\frac{3}{4}$ of an apple.

$\frac{3}{4}$ of an apple means that an apple has been divided into 4 equal parts and 3 of these parts taken.

In a fraction:

(1) The figure below the line is the denominator; it denominates or names the number of equal parts into which the unit is divided; it is the namer.

(2) The figure above the line is the numerator; it numerates or tells the number of parts taken; it is the numberer.

The numerator and denominator are the terms of the fraction.

1. In the fraction $\frac{5}{6}$:

(1) What is the 6 called? What does it show?

(2) What is the 5 called? What does it show?

(3) What does the fraction $\frac{5}{6}$ show?

2. Read aloud and tell what the terms of each fraction show:

$\frac{7}{9}$	$\frac{5}{12}$	$\frac{2}{7}$	$\frac{3}{8}$	$\frac{19}{20}$
$\$\frac{4}{5}$	$\frac{2}{3}$ of an acre	$\frac{3}{4}$ bu.	$\frac{7}{8}$ gal.	$\frac{5}{6}$ yd.

CHANGING THE FORM OF FRACTIONS *Oral*

1. Which do you prefer, $\frac{2}{4}$ of a dollar or $\frac{1}{2}$ of a dollar? $\frac{6}{8}$ of a pound of candy or $\frac{3}{4}$ of a pound of candy?

2. Are $\frac{2}{4}$ and $\frac{1}{2}$ alike in form? Are they alike in value?

3. Are $\frac{6}{8}$ and $\frac{3}{4}$ alike in value? In form?

4. How many equal parts in $\frac{3}{4}$ of a dollar?

5. How many equal parts in $\frac{6}{8}$ of a dollar?

6. What is the size of the equal parts in $\frac{3}{4}$ of a dollar?

7. What is the size of the equal parts in $\frac{6}{8}$ of a dollar?

8. Are 8ths of a dollar larger or smaller than 4ths of a dollar?

In $\frac{6}{8}$ of a dollar there are twice as many equal parts as in $\frac{3}{4}$ of a dollar, but the equal parts are only one half as large.

In $\frac{3}{4}$ of a dollar there are one half as many equal parts as in $\frac{6}{8}$ of a dollar, but the equal parts are twice as large.

The form of a fraction may be changed without changing its value:

(1) By multiplying both terms of the fraction by the same number.

Thus, $$\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$$

This is changing to higher terms.

(2) By dividing both terms of the fraction by the same number.

Thus, $$\frac{6}{8} \div \frac{2}{2} = \frac{3}{4}$$

This is changing to lower terms.

CHANGING TO HIGHER TERMS

1. Change $\frac{3}{8}$ to twenty-fourths.

$\frac{3}{8} \times \frac{3}{3} = \frac{9}{24}$ To get 24 as the denominator, we multiply the denominator by 3. In order not to change the value of the fraction, we also multiply the numerator by 3.

To change a fraction to higher terms, multiply both terms of the fraction by that number which will give the required denominator.

2. Change to 12ths: $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{5}{6}$
3. Change to 18ths: $\frac{2}{9}$ $\frac{2}{3}$ $\frac{1}{2}$ $\frac{1}{6}$ $\frac{5}{9}$ $\frac{5}{6}$
4. Change to 24ths: $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{5}{8}$ $\frac{7}{12}$
5. Change to 32ds: $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{8}$ $\frac{3}{16}$
6. Change to 36ths: $\frac{1}{18}$ $\frac{2}{9}$ $\frac{5}{12}$ $\frac{5}{6}$ $\frac{3}{4}$ $\frac{2}{3}$ $\frac{1}{2}$
7. Change to 40ths: $\frac{3}{20}$ $\frac{7}{10}$ $\frac{4}{5}$ $\frac{5}{8}$ $\frac{1}{2}$ $\frac{3}{4}$

CHANGING TO LOWEST TERMS *Oral*

1. Change $\frac{24}{54}$ to its lowest terms.

$$\frac{24 \div 2}{54 \div 2} = \frac{12 \div 3}{27 \div 3} = \frac{4}{9}$$

Dividing both terms of $\frac{24}{54}$ by 2, we get $\frac{12}{27}$; dividing both terms of $\frac{12}{27}$ by 3, we get $\frac{4}{9}$.

or

$$\frac{24 \div 6}{54 \div 6} = \frac{4}{9}$$

We can change the fraction more quickly by dividing both terms by their greatest common factor, 6.

A fraction is in its lowest terms when its numerator and denominator have no common factor.

Change to lowest terms:

2. $\frac{7}{35}$ $\frac{14}{30}$ $\frac{27}{33}$ $\frac{35}{40}$ $\frac{21}{36}$ $\frac{8}{30}$ $\frac{20}{28}$ $\frac{28}{32}$ $\frac{24}{36}$ $\frac{21}{42}$
3. $\frac{12}{30}$ $\frac{36}{36}$ $\frac{12}{40}$ $\frac{14}{42}$ $\frac{12}{44}$ $\frac{16}{32}$ $\frac{30}{42}$ $\frac{32}{36}$ $\frac{24}{40}$ $\frac{16}{44}$
4. $\frac{20}{45}$ $\frac{24}{32}$ $\frac{18}{36}$ $\frac{24}{60}$ $\frac{15}{45}$ $\frac{27}{42}$ $\frac{18}{45}$ $\frac{36}{40}$ $\frac{42}{44}$ $\frac{36}{48}$

Written

5. $\frac{40}{56}$ 6. $\frac{64}{72}$ 7. $\frac{54}{66}$ 8. $\frac{63}{69}$ 9. $\frac{36}{96}$
10. $\frac{70}{85}$ 11. $\frac{56}{84}$ 12. $\frac{52}{78}$ 13. $\frac{76}{84}$ 14. $\frac{56}{88}$
15. $\frac{69}{96}$ 16. $\frac{60}{105}$ 17. $\frac{84}{120}$ 18. $\frac{54}{276}$ 19. $\frac{99}{135}$
20. $\frac{96}{144}$ 21. $\frac{70}{315}$ 22. $\frac{256}{896}$ 23. $\frac{252}{280}$ 24. $\frac{128}{632}$
25. $\frac{112}{120}$ 26. $\frac{136}{276}$ 27. $\frac{216}{252}$ 28. $\frac{863}{1089}$ 29. $\frac{516}{1280}$

CHANGING IMPROPER FRACTIONS TO WHOLE OR MIXED NUMBERS

A fraction whose numerator is equal to or greater than its denominator is an improper fraction; as, $\frac{2}{2}$, $\frac{5}{5}$, $\frac{4}{3}$, $\frac{7}{5}$.

A number made up of a whole number and a fraction is a mixed number; as, $2\frac{1}{2}$, $3\frac{5}{8}$.

Oral

1. Change $\frac{22}{5}$ to a mixed number.

$\frac{22}{5} = 4\frac{2}{5}$ Since there are 5 fifths in one unit, in 22 fifths there are as many units as there are 5's in 22; that is, 4 units and 2 fifths of a unit.

Change to whole or mixed numbers:

2. $\frac{4}{4}$ $\frac{18}{5}$ $\frac{19}{7}$ $\frac{25}{5}$ $\frac{43}{8}$ $\frac{27}{4}$ $\frac{34}{3}$ $\frac{56}{7}$

3. $\frac{63}{10}$ $\frac{55}{6}$ $\frac{48}{8}$ $\frac{50}{7}$ $\frac{72}{12}$ $\frac{84}{7}$ $\frac{75}{8}$ $\frac{50}{11}$

4. $\frac{60}{2}$ pints $\frac{69}{8}$ yards $\frac{55}{7}$ weeks $\frac{45}{8}$ dollars $\$\frac{56}{8}$

5. $\frac{48}{8}$ bushels $\frac{85}{9}$ miles $\frac{60}{5}$ acres $\frac{87}{12}$ dozen $\$\frac{42}{5}$

Written

6. $\frac{567}{12}$ 7. $\frac{125}{8}$ 8. $\frac{120}{16}$ 9. $\frac{124}{32}$ bu. 10. $\frac{105}{24}$ days

11. $\frac{157}{9}$ 12. $\frac{107}{15}$ 13. $\frac{111}{18}$ 14. $\frac{165}{7}$ wks. 15. $\frac{223}{60}$ hours

16. $\frac{375}{32}$ 17. $\frac{157}{40}$ 18. $\frac{253}{25}$ 19. $\frac{168}{16}$ lb. 20. $\frac{125}{8}$ pecks

21. $\frac{250}{30}$ 22. $\frac{225}{17}$ 23. $\frac{382}{19}$ 24. $\$\frac{243}{50}$ 25. $\frac{647}{12}$ years

CHANGING WHOLE OR MIXED NUMBERS TO IMPROPER FRACTIONS

Oral

1. Change 2 to a fraction with 3 for a denominator.

$2 = \frac{2}{1} = \frac{6}{3}$

2. Change to halves: 1 3 5 6 8

3. Change to tenths: 1 4 6 8 10

4. Change to twentieths: 1 2 4 5 10

5. Change $4\frac{5}{8}$ to an improper fraction.

$4\frac{5}{8} = \frac{37}{8}$ $\quad 1 = \frac{8}{8}$; $4 = \frac{32}{8}$; $\frac{32}{8} + \frac{5}{8} = \frac{37}{8}$

Tell how to change a whole or mixed number to an improper fraction.

Change to improper fractions:

6. $2\frac{4}{5}$	7. $3\frac{3}{8}$	8. $2\frac{5}{7}$	9. $1\frac{5}{9}$	10. $7\frac{6}{7}$	11. $9\frac{5}{6}$
12. $8\frac{3}{4}$	13. $7\frac{2}{3}$	14. $5\frac{3}{10}$	15. $6\frac{7}{12}$	16. $8\frac{4}{11}$	17. $4\frac{7}{20}$

Written

18. $16\frac{1}{2}$	19. $15\frac{2}{3}$	20. $18\frac{3}{4}$	21. $16\frac{3}{5}$	22. $17\frac{1}{6}$
23. $24\frac{3}{8}$	24. $19\frac{4}{5}$	25. $26\frac{5}{8}$	26. $17\frac{5}{9}$	27. $20\frac{4}{7}$
28. $32\frac{5}{6}$	29. $48\frac{5}{9}$	30. $67\frac{7}{10}$	31. $37\frac{6}{11}$	32. $27\frac{5}{12}$
33. $45\frac{7}{12}$	34. $42\frac{8}{15}$	35. $55\frac{9}{16}$	36. $29\frac{11}{18}$	37. $45\frac{17}{24}$

REVIEW EXERCISE *Oral*

1. Change 1, 2, 3, 4, 5 to fractions with 8 for a denominator.

2. What are such fractions called?

3. Name five fractions each equal to 1.

4. Name five fractions each equal to $\frac{2}{3}$.

5. Name all the fractions equal to $\frac{1}{10}$ having two figures in the denominator.

6. Change to lowest terms: $\frac{2}{2}$ $\frac{2}{6}$ $\frac{10}{15}$ $\frac{15}{25}$ $\frac{12}{20}$ $\frac{18}{27}$ $\frac{9}{63}$ $\frac{12}{54}$ $\frac{70}{90}$ $\frac{36}{60}$ $\frac{35}{60}$ $\frac{42}{56}$ $\frac{54}{81}$ $\frac{60}{72}$

7. When is a fraction in its lowest terms?

8. Change to 20ths: $\frac{1}{2}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{7}{10}$ $\frac{6}{40}$ $\frac{20}{60}$ $\frac{24}{80}$ $\frac{25}{100}$

9. Change to 100ths: $\frac{1}{2}$ $\frac{3}{4}$ $\frac{3}{5}$ $\frac{7}{10}$ $\frac{9}{20}$ $\frac{4}{25}$ $\frac{11}{50}$

10. Changed to mixed numbers: $\frac{20}{3}$ $\frac{27}{8}$ $\frac{40}{7}$ $\frac{63}{8}$ $\frac{67}{12}$

11. Change to improper fractions: $3\frac{2}{5}$ $5\frac{3}{8}$ $7\frac{5}{9}$ $8\frac{4}{7}$ $5\frac{5}{12}$

ADDITION OF FRACTIONS *Oral*

1. Count by $\frac{1}{2}$'s to 6.
2. Count by $1\frac{1}{2}$'s to 15.

Thus, $\frac{1}{2}$, 1, $1\frac{1}{2}$, etc.

3. Count by $2\frac{1}{2}$'s to 30.
4. Count by $\frac{1}{4}$'s to 3.
5. Count by $1\frac{1}{4}$'s to 16.
6. Count by $2\frac{1}{4}$'s to 27.
7. Count by $\frac{1}{3}$'s to 4.
8. Count by $1\frac{1}{3}$'s to 15.
9. Count by $2\frac{1}{3}$'s to 28.
10. What change must be made before $\frac{1}{2}$ can be added to $\frac{1}{4}$? Why?
11. Add $\frac{1}{2}$ to: $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{8}$ $\frac{5}{8}$ $\frac{1}{10}$ $\frac{7}{10}$ $\frac{1}{12}$ $\frac{1}{16}$
12. Add $\frac{1}{3}$ to: $\frac{1}{6}$ $\frac{5}{6}$ $\frac{1}{9}$ $\frac{4}{9}$ $\frac{8}{9}$ $\frac{1}{12}$ $\frac{5}{12}$ $\frac{7}{15}$
13. Add $\frac{1}{4}$ to: $\frac{1}{8}$ $\frac{5}{8}$ $\frac{1}{12}$ $\frac{5}{12}$ $\frac{11}{12}$ $\frac{1}{16}$ $\frac{5}{16}$ $\frac{9}{16}$
14. Add $\frac{1}{5}$ to: $\frac{1}{10}$ $\frac{3}{10}$ $\frac{7}{10}$ $\frac{1}{15}$ $\frac{4}{15}$ $\frac{1}{20}$ $\frac{3}{20}$ $\frac{13}{20}$

Add:

15. $1\frac{1}{2}$, $3\frac{1}{2}$, $2\frac{1}{2}$
16. $2\frac{3}{4}$, $1\frac{1}{4}$, $5\frac{1}{4}$
17. $3\frac{1}{2}$, $1\frac{1}{4}$, $2\frac{1}{2}$
18. $2\frac{1}{2}$, $3\frac{3}{4}$, $2\frac{3}{4}$
19. $2\frac{1}{3}$, $1\frac{1}{2}$, $1\frac{1}{3}$
20. $2\frac{1}{5}$, $3\frac{7}{10}$, $2\frac{3}{5}$
21. $1\frac{5}{6}$, $2\frac{2}{3}$, $1\frac{1}{6}$
22. $3\frac{1}{3}$, $3\frac{1}{6}$, $3\frac{1}{2}$
23. $4\frac{2}{5}$, $3\frac{1}{2}$, $6\frac{4}{5}$
24. $7\frac{1}{2}$, $2\frac{2}{3}$, $2\frac{1}{2}$

Written

25. Add $\frac{3}{4}$, $\frac{2}{5}$, and $\frac{5}{12}$.

$c.\ d. = 60$

$\frac{3}{4} = \frac{45}{60}$

$\frac{2}{5} = \frac{24}{60}$

$\frac{5}{12} = \frac{25}{60}$

$\frac{94}{60} = 1\frac{34}{60} = 1\frac{17}{30}$

The least common multiple of the denominators 4, 5, and 12 is 60. Express $\frac{3}{4}$, $\frac{2}{5}$, and $\frac{5}{12}$ as 60ths.

Always change the fraction in the answer to its lowest terms.

To add fractions, change the fractions to equivalent fractions having a common denominator, and write the sum of the numerators over the common denominator.

Solve:

26. $\frac{3}{4} + \frac{7}{8}$ **27.** $\frac{8}{15} + \frac{4}{5}$ **28.** $\frac{2}{3} + 2\frac{5}{24}$ **29.** $2\frac{9}{20} + \frac{3}{4}$

30. $\frac{4}{5} + \frac{1}{2}$ **31.** $\frac{2}{5} + \frac{5}{6}$ **32.** $\frac{3}{8} + \frac{2}{3}$ **33.** $\frac{2}{3} + \frac{3}{4}$

34. $\frac{3}{4} + \frac{5}{6}$ **35.** $\frac{5}{6} + \frac{7}{9}$ **36.** $\frac{5}{12} + \frac{8}{9}$ **37.** $\frac{5}{8} + \frac{7}{12}$

38. $\frac{7}{10} + \frac{3}{4}$ **39.** $\frac{8}{10} + \frac{5}{6}$ **40.** $\frac{7}{12} + \frac{3}{20}$ **41.** $\frac{5}{16} + \frac{7}{12}$

42. $\frac{2}{9} + \frac{3}{8}$ **43.** $\frac{5}{12} + \frac{7}{15}$ **44.** $\frac{7}{12} + \frac{5}{18}$ **45.** $\frac{9}{20} + \frac{8}{15}$

46. $\frac{1}{2} + \frac{3}{8} + \frac{7}{16}$ **47.** $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{9}{16}$

48. $\frac{1}{6} + \frac{5}{18} + \frac{2}{9}$ **49.** $\frac{1}{2} + \frac{1}{3} + \frac{3}{4} + \frac{5}{6}$

50. $\frac{5}{8} + \frac{2}{3} + \frac{5}{6}$ **51.** $\frac{1}{2} + \frac{2}{3} + \frac{5}{6} + \frac{5}{9}$

52. $\frac{3}{4} + \frac{4}{5} + \frac{7}{10}$ **53.** $\frac{2}{3} + \frac{3}{4} + \frac{5}{8} + \frac{1}{6}$

54. $\frac{7}{9} + \frac{1}{2} + \frac{3}{4}$ **55.** $\frac{2}{3} + \frac{3}{5} + \frac{5}{6} + \frac{7}{10}$

ADDITION OF MIXED NUMBERS *Written*

1. Add $8\frac{5}{6}$ and $2\frac{3}{8}$.

$$\begin{array}{r} c.\ d. = 24 \\ \hline 8\frac{5}{6} = 8\frac{20}{24} \\ 2\frac{3}{8} = 2\frac{9}{24} \\ \hline 11\frac{5}{24} \end{array}$$

Changing $\frac{5}{6}$ and $\frac{3}{8}$ to similar fractions, we get $\frac{20}{24}$ and $\frac{9}{24}$. $\frac{20}{24} + \frac{9}{24} = \frac{29}{24} = 1\frac{5}{24}$. Write the $\frac{5}{24}$ and add 1 to the units.

Solve:

2. $2\frac{5}{12} + 3\frac{3}{4}$ **3.** $2\frac{5}{6} + 8\frac{1}{4}$ **4.** $3\frac{4}{9} + 7\frac{5}{12}$ **5.** $5\frac{3}{10} + 7\frac{8}{15}$

6. $7\frac{5}{8} + 4\frac{9}{20}$ **7.** $6\frac{8}{9} + 9\frac{7}{12}$ **8.** $5\frac{3}{8} + 4\frac{2}{5}$ **9.** $6\frac{5}{8} + 7\frac{7}{12}$

10. $7\frac{5}{12} + 8\frac{4}{15}$ **11.** $8\frac{3}{10} + 8\frac{7}{15}$ **12.** $9\frac{6}{7} + 8\frac{2}{3}$ **13.** $5\frac{4}{5} + 2\frac{3}{7}$

14. $4\frac{3}{7} + 5\frac{3}{4}$ **15.** $6\frac{7}{10} + 5\frac{4}{25}$ **16.** $8\frac{3}{8} + 6\frac{7}{10}$ **17.** $9\frac{4}{15} + 8\frac{7}{20}$

18. $\$12\frac{4}{5} + \$3\frac{7}{10} + \$18\frac{9}{20}$ **19.** $4\frac{3}{4} + 7\frac{2}{3} + 8\frac{5}{8}$

20. $15\frac{3}{4} + 20\frac{5}{8} + 10\frac{7}{16}$ **21.** $12\frac{2}{3} + 3\frac{4}{9} + 2\frac{7}{15} + 4\frac{2}{5}$

22. $2\frac{3}{5} + 8\frac{7}{15} + 10\frac{1}{2} + 15\frac{2}{3}$ **23.** $1\frac{3}{4} + 2\frac{7}{8} + 4\frac{5}{16} + 6\frac{1}{2}$

DRILL EXERCISE *Oral and Written*

	A	*B*	*C*	*D*
1.	$\frac{5}{6}+\frac{1}{2}$	$\frac{5}{12}+\frac{3}{4}$	$\frac{1}{2}+\frac{7}{10}$	$\frac{4}{5}+\frac{2}{3}$
2.	$\frac{2}{3}+\frac{3}{4}$	$\frac{2}{5}+\frac{1}{2}$	$\frac{3}{4}+\frac{7}{9}$	$\frac{5}{8}+\frac{4}{5}$
3.	$\frac{3}{4}+\frac{4}{5}$	$\frac{1}{2}+\frac{5}{7}$	$\frac{2}{5}+\frac{5}{6}$	$\frac{3}{8}+\frac{5}{8}$
4.	$\frac{1}{4}+\frac{5}{6}$	$\frac{3}{8}+\frac{1}{6}$	$\frac{5}{8}+\frac{3}{10}$	$\frac{1}{6}+\frac{8}{9}$
5.	$\frac{7}{15}+\frac{3}{20}$	$\frac{3}{4}+\frac{7}{10}$	$\frac{2}{3}+\frac{4}{5}$	$\frac{5}{8}+\frac{7}{12}$
6.	$\frac{7}{12}+\frac{8}{15}$	$\frac{5}{6}+\frac{7}{15}$	$\frac{1}{6}+\frac{1}{14}$	$\frac{5}{6}+\frac{7}{20}$
7.	$3\frac{7}{10}+\frac{2}{3}$	$2\frac{5}{6}+\frac{3}{4}$	$3\frac{3}{4}+3\frac{2}{3}$	$4\frac{2}{3}+5\frac{4}{9}$
8.	$1\frac{3}{10}+1\frac{8}{15}$	$1\frac{5}{9}+3\frac{7}{12}$	$5\frac{5}{6}+4\frac{7}{10}$	$3\frac{2}{7}+5\frac{2}{14}$
9.	$3\frac{5}{12}+4\frac{7}{15}$	$2\frac{3}{4}+5\frac{7}{9}$	$2\frac{9}{10}+3\frac{4}{25}$	$3\frac{5}{12}+4\frac{7}{20}$
10.	$1\frac{3}{20}+6\frac{5}{24}$	$2\frac{7}{12}+4\frac{8}{27}$	$6\frac{4}{9}+3\frac{11}{12}$	$4\frac{5}{18}+2\frac{17}{20}$

PROBLEMS *Oral and Written*

1. James sold $\frac{1}{3}$ of a box of corncakes one day and $\frac{1}{4}$ of a box the next. What part of a box on both days?

2. The charge for one telephone message was $\$\frac{2}{5}$ and for another $\$\frac{1}{4}$. What part of a dollar did both cost?

3. Mrs. Jenkins canned $\frac{2}{3}$ of a crate of blueberries on Tuesday and $\frac{1}{2}$ of a crate on Wednesday. How many crates did she can in all?

4. Henry walked $1\frac{1}{2}$ miles in going to school, $\frac{3}{4}$ of a mile on an errand, and $2\frac{1}{4}$ miles in the woods. How many miles in all?

5. Susie bought $\frac{1}{6}$ of a dozen oranges, Mary $\frac{1}{3}$ of a dozen, and Harriet $\frac{1}{4}$ of a dozen. What part of a dozen did they all buy?

6. How much must I pay for a pair of shoes at $\$3\frac{1}{2}$, a pair of rubbers at $\$\frac{4}{5}$, and a rubber coat at $\$5\frac{3}{4}$?

7. Chester put in his barn an electric bell to be rung from the house. The bell cost $\$\frac{2}{5}$, the wire $\$\frac{1}{4}$, the battery $\$\frac{1}{2}$, and the button $\$\frac{1}{10}$. How much did he pay for materials?

8. Last year Mr. Stone used $5\frac{3}{8}$ cords of wood, and this year $1\frac{1}{2}$ cords more. How many cords this year? How many cords in both years?

SUBTRACTION OF FRACTIONS

Oral

1. Take $\frac{1}{2}$ from: 1 $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{9}{14}$ $\frac{11}{16}$
2. Take $\frac{1}{3}$ from: 1 $\frac{5}{6}$ $\frac{4}{9}$ $\frac{7}{9}$ $\frac{5}{12}$ $\frac{11}{12}$ $\frac{7}{15}$ $\frac{5}{18}$
3. Take $\frac{1}{4}$ from: 1 $\frac{5}{8}$ $\frac{7}{8}$ $\frac{7}{12}$ $\frac{11}{12}$ $\frac{7}{16}$ $\frac{15}{16}$ $\frac{11}{20}$
4. Take $\frac{1}{5}$ from: 1 $\frac{8}{10}$ $\frac{7}{10}$ $\frac{8}{15}$ $\frac{11}{15}$ $\frac{7}{20}$ $\frac{9}{20}$ $\frac{17}{20}$
5. From 10 take:

$\frac{3}{8}$ $\frac{2}{9}$ $\frac{7}{10}$ $\frac{5}{12}$ $1\frac{3}{4}$ $2\frac{2}{3}$ $3\frac{4}{5}$ $3\frac{5}{6}$ $4\frac{6}{7}$ $6\frac{7}{8}$

6. Subtract $\frac{7}{10}$ from $\frac{5}{6}$.

c. d. = 30

$\frac{5}{6} = \frac{25}{30}$

$\frac{7}{10} = \frac{21}{30}$

$\frac{4}{30} = \frac{2}{15}$

Express $\frac{5}{6}$ and $\frac{7}{10}$ as similar fractions. $\frac{25}{30} - \frac{21}{30} = \frac{4}{30}$; $\frac{4}{30} = \frac{2}{15}$.

To subtract fractions, change the fractions to equivalent fractions having a common denominator, and write the difference of the numerators over the common denominator.

Solve: *Written*

7. $\frac{7}{8} - \frac{5}{6}$	8. $\frac{4}{9} - \frac{5}{18}$	9. $\frac{11}{15} - \frac{3}{5}$	10. $\frac{5}{8} - \frac{5}{24}$
11. $\frac{2}{3} - \frac{3}{10}$	12. $\frac{9}{10} - \frac{5}{6}$	13. $\frac{8}{9} - \frac{3}{4}$	14. $\frac{5}{6} - \frac{3}{5}$
15. $\frac{4}{5} - \frac{3}{8}$	16. $\frac{7}{8} - \frac{3}{10}$	17. $\frac{8}{9} - \frac{5}{6}$	18. $\frac{7}{10} - \frac{1}{6}$
19. $\frac{3}{4} - \frac{3}{10}$	20. $\frac{11}{12} - \frac{5}{8}$	21. $\frac{7}{12} - \frac{5}{9}$	22. $\frac{1}{15} - \frac{1}{20}$
23. $\frac{7}{8} - \frac{1}{6}$	24. $\frac{9}{10} - \frac{5}{12}$	25. $\frac{11}{16} - \frac{5}{12}$	26. $\frac{5}{12} - \frac{7}{18}$

SUBTRACTION OF MIXED NUMBERS *Written*

1. Subtract $3\frac{5}{6}$ from $8\frac{8}{10}$.

$c.\ d. = 30$

$8\frac{8}{10} = 8\frac{24}{30} = 7\frac{54}{30}$

$3\frac{5}{6} = \quad = 3\frac{25}{30}$

$4\frac{29}{30} = 4\frac{7}{15}$

Expressing the fractions $\frac{8}{10}$ and $\frac{5}{6}$ as similar fractions, we get $\frac{24}{30}$ and $\frac{25}{30}$. Since $\frac{25}{30}$ cannot be taken from $\frac{24}{30}$, we take 1 from the 8, express it as $\frac{30}{30}$, and add it to the $\frac{24}{30}$, making $\frac{54}{30}$. $\frac{54}{30} - \frac{25}{30} = \frac{29}{30}$, or $\frac{7}{15}$.

Solve:

2. $16\frac{5}{8} - 8\frac{3}{4}$ **3.** $12\frac{1}{4} - 3\frac{2}{5}$ **4.** $12\frac{1}{5} - 3\frac{1}{2}$

5. $15\frac{2}{3} - 3\frac{2}{5}$ **6.** $12\frac{5}{9} - 8\frac{5}{8}$ **7.** $17\frac{1}{6} - 5\frac{1}{8}$

8. $33\frac{7}{15} - 2\frac{8}{20}$ **9.** $48\frac{3}{4} - 24\frac{4}{5}$ **10.** $12\frac{5}{16} - 6\frac{5}{18}$

11. $18\frac{9}{20} - 3\frac{5}{8}$ **12.** $17\frac{6}{25} - 8\frac{7}{10}$ **13.** $27\frac{5}{12} - 2\frac{5}{12}$

14. $10\frac{2}{7} - 3\frac{3}{4}$ **15.** $25\frac{7}{8} - 10\frac{7}{10}$ **16.** $16\frac{5}{12} - 14\frac{7}{8}$

17. Which is the shorter distance, $\frac{5}{6}$ of a mile or $\frac{7}{8}$ of a mile? How much shorter?

18. What must be taken from $4\frac{1}{6}$ to leave $2\frac{2}{3}$?

19. What must be added to $5\frac{2}{3}$ to make $12\frac{3}{8}$?

20. $\$12\frac{4}{5}$ is how much more than $\$8\frac{3}{4}$?

21. Change $\frac{42}{56}$ and $\frac{63}{72}$ to lowest terms and find their difference.

PROBLEMS *Oral and Written*

1. After selling $\frac{5}{16}$ of a farm, what part is left?

2. Ralph picked $\frac{5}{8}$ of a peck of cherries. He sold $\frac{1}{2}$ of a peck. What part of a peck had he then?

3. Isabel had $\$\frac{4}{5}$. She bought a belt for $\$\frac{1}{2}$. What part of a dollar had she left?

4. What part of a dollar did John lose by selling for $\$\frac{1}{4}$ a knife which cost $\$\frac{3}{5}$?

5. Roy earned $\$1\frac{1}{4}$ and spent $\$\frac{1}{2}$. What part of a dollar had he left?

6. One half of Mr. Arbin's land is a vegetable garden, $\frac{1}{8}$ is a flower garden, and the rest is in lawn. What part in lawn?

7. After selling $\frac{3}{4}$ of a bushel and $\frac{1}{2}$ of a bushel, what is left of a barrel of apples ($2\frac{1}{2}$ bu.)?

8. Gladys is $12\frac{1}{6}$ years old, and her sister is $2\frac{1}{2}$ years younger. What is the sister's age?

9. One fourth of a farm is in pasture, $\frac{2}{5}$ in grass, and the rest in grain. What part in grain?

10. Harry earned \$3 a week for 5 weeks. He then bought a suit for $\$12\frac{3}{4}$. How much had he left?

DRILL EXERCISE *Oral or Written*

	A	*B*	*C*	*D*
1.	$\frac{1}{2}+?=\frac{3}{4}$	$?+\frac{2}{3}=\frac{7}{8}$	$\frac{3}{4}-?=\frac{1}{2}$	$?-\frac{3}{4}=\frac{1}{2}$
2.	$\frac{2}{3}+?=\frac{5}{6}$	$?+\frac{1}{2}=\frac{5}{6}$	$\frac{5}{8}-?=\frac{1}{4}$	$?-\frac{5}{8}=\frac{1}{4}$
3.	$\frac{1}{4}+?=\frac{5}{12}$	$?+\frac{3}{4}=\frac{4}{5}$	$\frac{5}{6}-?=\frac{2}{3}$	$?-\frac{5}{6}=\frac{2}{3}$
4.	$\frac{2}{5}+?=\frac{5}{8}$	$?+\frac{3}{8}=\frac{2}{3}$	$\frac{3}{5}-?=\frac{1}{2}$	$?-\frac{3}{5}=\frac{1}{2}$
5.	$\frac{5}{6}+?=1\frac{1}{2}$	$?+\frac{4}{5}=2$	$1\frac{1}{5}-?=\frac{2}{5}$	$?-1\frac{1}{5}=\frac{2}{5}$
6.	$\frac{3}{8}+?=2$	$?+\frac{3}{5}=1$	$2-?=1\frac{1}{4}$	$?-2=1\frac{1}{4}$
7.	$\frac{7}{9}+?=1\frac{2}{3}$	$?+2\frac{1}{3}=3\frac{1}{4}$	$1\frac{1}{4}-?=\frac{7}{8}$	$?-1\frac{1}{4}=\frac{7}{8}$
8.	$2\frac{1}{5}+?=5\frac{1}{2}$	$?+5\frac{1}{2}=6\frac{1}{4}$	$3\frac{1}{4}-?=2\frac{1}{5}$	$?-3\frac{1}{4}=2\frac{1}{5}$
9.	$1\frac{3}{8}+?=2\frac{5}{6}$	$?+1\frac{3}{4}=2\frac{3}{8}$	$2\frac{3}{8}-?=1\frac{3}{4}$	$?-2\frac{3}{8}=1\frac{3}{4}$
10.	$1\frac{5}{8}+?=3\frac{1}{2}$	$?+1\frac{5}{8}=2\frac{1}{4}$	$3\frac{2}{3}-?=2\frac{1}{6}$	$?-3\frac{2}{3}=2\frac{1}{6}$

PROBLEMS *Oral and Written*

1. A kerosene can holds 5 gallons. How much is left after $1\frac{3}{4}$ gallons have been drawn?

2. Mrs. Emerson paid $\$\frac{3}{5}$ for eggs and $\$\frac{3}{4}$ for sugar. How much for both?

3. A man's step is $2\frac{5}{12}$ feet, and his son's $1\frac{3}{8}$ feet. How much longer than the son's step is the father's?

4. A carpenter has a board $12\frac{2}{3}$ feet long with which to cover a space $10\frac{7}{12}$ feet long. How much must he cut off?

5. Albert spent $\frac{1}{2}$ of his money for a ball and $\frac{1}{5}$ of it for a bat. What part of his money had he left? How much had he left if he had 60 cents at first?

6. A tub of butter weighs $56\frac{1}{2}$ pounds. The tub weighs $3\frac{9}{16}$ pounds. What is the weight of the butter?

7. A man can saw $\frac{1}{2}$ of a pile of wood in a day, and a boy can saw $\frac{1}{3}$ of it. What part can both saw in a day?

8. In a storm at sea, $\frac{1}{4}$ of a cargo of fruit was thrown overboard and $\frac{2}{5}$ was spoiled. What part of the cargo was lost? What part was saved?

9. What is the perimeter of an envelope, $6\frac{1}{2}$ inches long and $3\frac{3}{4}$ inches wide?

10. A seam $\frac{3}{16}$ of an inch wide is made on both sides of a strip of cloth 27 inches wide. What is the width of the strip after the seams are made?

11. A bat cost $\$\frac{3}{10}$, a ball $\$\frac{4}{5}$, and a pair of ball shoes $\$1\frac{3}{4}$. What was the change from a $\$5$-bill?

12. From a piece of dress goods containing 60 yards there were sold $12\frac{2}{3}$ yards, $8\frac{5}{6}$ yards, $11\frac{7}{8}$ yards, and $19\frac{3}{4}$ yards. How many yards were left?

13. Spice bought at $27\frac{3}{4}$ cents a pound is sold at 42 cents a pound. What is the gain?

14. Coffee is sold for 38 cents a pound. The gain is $8\frac{3}{5}$ cents. What was the cost?

MULTIPLICATION OF FRACTIONS *Oral*

1. 6 times 3 apples = ? **2.** 6 times 3 sevenths = ?

3. $6 \times \frac{3}{7} = 1\frac{8}{7} = 2\frac{4}{7}$.

Remember that the denominator simply names the fraction.

Solve:

4. $7 \times \frac{1}{4}$	**5.** $6 \times \frac{2}{5}$	**6.** $5 \times \frac{3}{4}$	**7.** $8 \times \frac{2}{3}$
8. $4 \times \frac{3}{7}$	**9.** $10 \times \frac{2}{3}$	**10.** $4 \times \frac{3}{5}$	**11.** $20 \times \frac{1}{3}$
12. $8 \times \frac{3}{5}$	**13.** $7 \times \frac{5}{6}$	**14.** $6 \times \frac{5}{7}$	**15.** $8 \times \frac{1}{5}$
16. $7 \times \frac{2}{9}$	**17.** $6 \times \frac{4}{7}$	**18.** $7 \times \frac{2}{3}$	**19.** $12 \times \frac{2}{5}$
20. $5 \times \frac{3}{8}$	**21.** $10 \times \frac{4}{9}$	**22.** $7 \times \frac{5}{8}$	**23.** $16 \times \frac{2}{3}$

Written

24. $15 \times \frac{3}{4}$	**25.** $29 \times \frac{2}{3}$	**26.** $20 \times \frac{6}{7}$	**27.** $17 \times \frac{8}{15}$
28. $28 \times \frac{1}{3}$	**29.** $25 \times \frac{5}{6}$	**30.** $24 \times \frac{8}{11}$	**31.** $24 \times \frac{3}{5}$
32. $18 \times \frac{4}{5}$	**33.** $27 \times \frac{3}{16}$	**34.** $45 \times \frac{7}{8}$	**35.** $37 \times \frac{7}{12}$
36. $11 \times \frac{5}{14}$	**37.** $32 \times \frac{7}{15}$	**38.** $27 \times \frac{5}{8}$	**39.** $35 \times \frac{9}{16}$
40. $14 \times \frac{7}{9}$	**41.** $25 \times \frac{5}{12}$	**42.** $40 \times \frac{4}{11}$	**43.** $43 \times \frac{8}{9}$

MULTIPLYING A WHOLE NUMBER BY A FRACTION *Oral*

1. Find $\frac{3}{5} \times 40$.

In the expression $\frac{3}{5} \times 8$ the sign $\times$ is equivalent to the word "of."

Find:

2. $\frac{3}{8} \times 24$	**3.** $\frac{3}{7} \times 42$	**4.** $\frac{6}{11} \times 33$	**5.** $\frac{5}{9} \times 27$
6. $\frac{2}{3} \times 18$	**7.** $\frac{6}{7} \times 84$	**8.** $\frac{5}{6} \times 72$	**9.** $\frac{5}{12} \times 144$
10. $\frac{4}{5} \times 50$	**11.** $\frac{5}{8} \times 56$	**12.** $\frac{7}{9} \times 63$	**13.** $\frac{5}{8} \times 54$

Finding a fractional part of a number is called multiplying by a fraction.

14. $\frac{2}{5} \times 8$

$\frac{1}{5}$ of $8 = \frac{8}{5}$; $\frac{2}{5}$ of $8 = \frac{16}{5} = 3\frac{1}{5}$.

15. $\frac{2}{3} \times 7$	**16.** $\frac{3}{8} \times 5$	**17.** $\frac{3}{5} \times 8$	**18.** $\frac{3}{8} \times 11$
19. $\frac{3}{4} \times 9$	**20.** $\frac{3}{5} \times 3$	**21.** $\frac{2}{3} \times 4$	**22.** $\frac{2}{9} \times 8$
23. $\frac{4}{5} \times 6$	**24.** $\frac{1}{7} \times 8$	**25.** $\frac{3}{11} \times 9$	**26.** $\frac{3}{4} \times 9$
27. $\frac{5}{6} \times 7$	**28.** $\frac{3}{10} \times 7$	**29.** $\frac{5}{12} \times 5$	**30.** $\frac{5}{8} \times 7$
31. $\frac{3}{7} \times 6$	**32.** $\frac{5}{11} \times 9$	**33.** $\frac{7}{12} \times 11$	**34.** $\frac{8}{9} \times 4$

Written

35. $\frac{1}{2} \times 45$	**36.** $\frac{2}{3} \times 25$	**37.** $\frac{3}{5} \times 19$	**38.** $\frac{2}{7} \times 32$
39. $\frac{3}{4} \times 19$	**40.** $\frac{5}{8} \times 15$	**41.** $\frac{7}{9} \times 40$	**42.** $\frac{3}{8} \times 37$
43. $\frac{5}{6} \times 23$	**44.** $\frac{4}{7} \times 30$	**45.** $\frac{3}{10} \times 39$	**46.** $\frac{5}{12} \times 41$
47. $\frac{6}{7} \times 45$	**48.** $\frac{4}{9} \times 50$	**49.** $\frac{5}{6} \times 53$	**50.** $\frac{8}{9} \times 64$
51. $\frac{7}{8} \times 81$	**52.** $\frac{7}{12} \times 71$	**53.** $\frac{5}{7} \times 80$	**54.** $\frac{7}{10} \times 83$

MULTIPLYING A FRACTION BY A FRACTION

1. $\frac{2}{3}$ of $\frac{3}{\text{days}} = ?$

$\frac{1}{3}$ of $\frac{3}{\text{days}} = \frac{1}{\text{day}}$

$\frac{2}{3}$ of $\frac{3}{\text{days}} = \frac{2}{\text{days}}$

2. $\frac{2}{3}$ of $\frac{3}{7} = ?$

$\frac{1}{3}$ of $\frac{3}{7} = \frac{1}{7}$

$\frac{2}{3}$ of $\frac{3}{7} = \frac{2}{7}$

If in finding the product of $\frac{2}{3} \times \frac{3}{7}$, the numerators are multiplied together for a new numerator, and the denominators multiplied together for a new denominator, the same result is obtained as in the process just described.

(1) $\frac{2}{3} \times \frac{3}{7} = \frac{6}{21} = \frac{2}{7}$ (2) $\frac{2}{\cancel{3}} \times \frac{\cancel{3}}{7} = \frac{2}{7}$

To multiply a fraction by a fraction, write the product of the numerators over the product of the denominators, canceling when possible.

This rule applies to all cases of multiplication in fractions, for every whole number may be written as a fraction with 1 for its denominator. $\frac{4}{5} \times 15$ may be written $\frac{4}{5} \times \frac{15}{1}$; $15 \times \frac{2}{3} = \frac{15}{1} \times \frac{2}{3}$.

Oral

Solve:

3. $\frac{2}{3} \times \frac{3}{4}$	4. $\frac{1}{3} \times \frac{15}{16}$	5. $\frac{4}{5} \times \frac{25}{4}$	6. $\frac{5}{8} \times \frac{4}{5}$
7. $\frac{5}{6} \times \frac{12}{15}$	8. $\frac{3}{4} \times \frac{8}{9}$	9. $\frac{2}{3} \times \frac{6}{7}$	10. $\frac{4}{5} \times \frac{5}{9}$
11. $\frac{2}{3} \times \frac{15}{16}$	12. $\frac{5}{6} \times \frac{18}{25}$	13. $\frac{7}{8} \times \frac{15}{16}$	14. $\frac{3}{4} \times \frac{20}{21}$
15. $\frac{3}{7} \times \frac{42}{63}$	16. $\frac{16}{21} \times \frac{3}{4}$	17. $\frac{5}{8} \times \frac{24}{25}$	18. $\frac{8}{9} \times \frac{27}{32}$
19. $\frac{22}{45} \times \frac{9}{11}$	20. $\frac{15}{17} \times \frac{17}{30}$	21. $\frac{8}{9} \times \frac{7}{24}$	22. $\frac{5}{12} \times \frac{15}{20}$
23. $\frac{4}{5} \times \frac{8}{15}$	24. $\frac{6}{9} \times \frac{17}{18}$	25. $\frac{4}{9} \times \frac{5}{6}$	26. $\frac{7}{8} \times \frac{9}{10}$

27. $2\frac{2}{3} \times 3\frac{3}{4} = \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{3}}} \times \frac{\overset{5}{\cancel{15}}}{\underset{1}{\cancel{4}}} = 10$

Written

28. $2\frac{1}{4} \times 2\frac{1}{3}$	29. $3\frac{3}{5} \times 1\frac{7}{9}$	30. $3\frac{1}{3} \times 4\frac{4}{5}$	31. $2\frac{3}{4} \times 1\frac{5}{11}$
32. $5\frac{5}{6} \times 4\frac{3}{7}$	33. $5\frac{1}{7} \times 1\frac{5}{9}$	34. $1\frac{3}{5} \times 2\frac{1}{3}$	35. $1\frac{9}{16} \times 1\frac{16}{25}$
36. $5\frac{2}{5} \times 1\frac{8}{9}$	37. $3\frac{1}{8} \times 4\frac{4}{5}$	38. $5\frac{1}{7} \times 1\frac{1}{3}$	39. $4\frac{3}{8} \times 2\frac{3}{7}$
40. $3\frac{1}{4} \times 2\frac{2}{3}$	41. $5\frac{3}{5} \times 1\frac{11}{14}$	42. $5\frac{5}{8} \times 1\frac{1}{2}$	43. $4\frac{2}{3} \times 2\frac{1}{7}$
44. $1\frac{5}{12} \times 4\frac{4}{5}$	45. $1\frac{7}{9} \times 5\frac{1}{4}$	46. $3\frac{5}{9} \times 2\frac{1}{3}$	47. $2\frac{6}{7} \times 5\frac{3}{5}$

48. Multiply $3\frac{3}{4}$ by 6.

$3\frac{3}{4}$

$\overset{3}{\cancel{6}}$

$4\frac{1}{2} = 6 \times \frac{3}{4}$

$18 = 6 \times 3$

$22\frac{1}{2}$

49. Multiply 6 by $3\frac{3}{4}$.

6

$3\frac{3}{4}$

$4\frac{1}{2} = \frac{3}{4} \times 6$

$18 = 3 \times 6$

$22\frac{1}{2}$

In multiplying a mixed number by a whole number, or a whole number by a mixed number, we may multiply without changing the mixed number to an improper fraction.

Give products at sight:

50. $4 \times 2\frac{1}{2}$ **51.** $6\frac{1}{2} \times 3$ **52.** $3 \times 2\frac{2}{3}$ **53.** $1\frac{1}{4} \times 7$

54. $5 \times 1\frac{1}{3}$ **55.** $1\frac{2}{3} \times 4$ **56.** $4 \times 1\frac{5}{7}$ **57.** $1\frac{3}{5} \times 4$

58. $6 \times 2\frac{3}{9}$ **59.** $2\frac{3}{5} \times 3$ **60.** $2 \times 2\frac{2}{5}$ **61.** $1\frac{1}{6} \times 3$

Written

Find:

62. $5 \times 3\frac{2}{5}$ **63.** $12\frac{3}{8} \times 5$ **64.** $9 \times 8\frac{5}{6}$ **65.** $7\frac{5}{8} \times 12$

66. $12 \times 3\frac{5}{6}$ **67.** $8\frac{4}{5} \times 10$ **68.** $8 \times 5\frac{5}{7}$ **69.** $13\frac{3}{5} \times 9$

70. $9 \times 16\frac{3}{10}$ **71.** $14\frac{5}{8} \times 16$ **72.** $13 \times 6\frac{8}{9}$ **73.** $14\frac{3}{8} \times 22$

74. $15 \times 22\frac{2}{8}$ **75.** $20\frac{5}{8} \times 12$ **76.** $7 \times 16\frac{2}{3}$ **77.** $12\frac{5}{6} \times 18$

78. $40 \times 14\frac{3}{4}$ **79.** $15\frac{5}{7} \times 28$ **80.** $24 \times 15\frac{7}{8}$ **81.** $81\frac{5}{12} \times 84$

DRILL EXERCISES *Oral and Written*

1. $\frac{1}{5}$ of $\frac{1}{8}$ $\frac{1}{5}$ of $\frac{7}{8}$ $\frac{2}{5}$ of $\frac{7}{8}$ $\frac{4}{5}$ of $\frac{5}{18}$ $\frac{2}{3}$ of 8

2. $\frac{1}{7}$ of $\frac{1}{5}$ $\frac{1}{7}$ of $\frac{3}{5}$ $\frac{2}{7}$ of $\frac{3}{5}$ $\frac{3}{8}$ of $\frac{3}{4}$ $\frac{3}{4}$ of $\frac{5}{8}$

3. $\frac{1}{9}$ of 27 $\frac{1}{9}$ of $\frac{27}{28}$ $\frac{4}{9}$ of $\frac{27}{28}$ $\frac{5}{6}$ of $\frac{21}{40}$ $\frac{5}{6}$ of $\frac{9}{20}$

4. $\frac{1}{6}$ of 12 $\frac{1}{6}$ of $\frac{12}{13}$ $\frac{5}{6}$ of $\frac{12}{13}$ $\frac{2}{9}$ of $\frac{15}{16}$ $\frac{6}{7}$ of $\frac{35}{36}$

5. $\frac{1}{8}$ of 11 $\frac{1}{8}$ of $\frac{11}{5}$ $\frac{5}{8}$ of $\frac{11}{5}$ $\frac{5}{11}$ of $\frac{44}{45}$ $\frac{7}{8}$ of $\frac{20}{63}$

6. $\frac{4}{5}$ of 20 $\frac{4}{5}$ of $\frac{20}{21}$ $\frac{3}{5}$ of $\frac{40}{81}$ $\frac{3}{8}$ of $\frac{1}{2}$ $\frac{8}{9}$ of 15

7. $\frac{2}{7}$ of $\frac{1}{8}$ $\frac{2}{7}$ of $\frac{21}{8}$ $\frac{5}{7}$ of $\frac{21}{35}$ $\frac{7}{12}$ of $\frac{48}{84}$ $\frac{9}{10}$ of $\frac{35}{36}$

8. $\frac{1}{3}$ of 11 $\frac{1}{3}$ of $\frac{1}{2}$ $\frac{2}{3}$ of 1 $\frac{2}{3}$ of $\frac{1}{2}$ $\frac{11}{12}$ of $\frac{12}{19}$

9. $\frac{1}{7}$ of 15 $\frac{1}{7}$ of 4 $\frac{1}{4}$ of 7 $\frac{3}{4}$ of $\frac{7}{15}$ $\frac{12}{13}$ of $\frac{39}{40}$

10. $\frac{5}{12}$ of $\frac{36}{75}$ $\frac{9}{13}$ of $\frac{65}{81}$ $\frac{8}{15}$ of $\frac{27}{32}$ $\frac{4}{7}$ of 8 $\frac{13}{14}$ of $\frac{42}{65}$

PROBLEMS *Oral and Written*

1. How much must be paid for $\frac{7}{8}$ of a pound of steak at 32 cents a pound?

2. How many days in $\frac{5}{6}$ of the month of September?

3. A chair cost \$12 and a table $2\frac{3}{8}$ times as much. How much did the table cost?

4. Mabel had \$$\frac{4}{5}$. She spent $\frac{5}{8}$ of it for a Dutch collar. What part of a dollar did the collar cost?

5. A street vender bought $2\frac{1}{2}$ bushels of chestnuts, and sold $\frac{2}{5}$ at once. What quantity did he sell?

6. A man earns \$$2\frac{2}{5}$ a day; how much in a week?

7. A farmer planted $3\frac{3}{4}$ acres to corn and $1\frac{1}{3}$ times as many acres to potatoes. How many acres to potatoes?

8. A dressmaker charged \$$5\frac{1}{4}$ for her labor, and $\frac{3}{7}$ as much for trimmings. How much for both?

9. The frames for my eyeglasses cost \$$4\frac{1}{2}$, and each of the lenses \$$1\frac{1}{4}$. What was the total cost?

10. After selling $\frac{3}{7}$ of a barrel of vinegar ($31\frac{1}{2}$ gallons), how many gallons has the grocer left?

DIVISION OF FRACTIONS

1. $\frac{6 \text{ apples}}{} \div \frac{2 \text{ apples}}{} = ?$

2. $\frac{6 \text{ sevenths}}{} \div \frac{2 \text{ sevenths}}{} = ?$

3. $\frac{6}{7} \div \frac{2}{7} = ?$

4. $\frac{6}{7} \div \frac{3}{7} = ?$

5. $\frac{6}{7} \div \frac{4}{7} = ?$

6. $\frac{6}{7} \div \frac{5}{7} = ?$

7. $\frac{3}{4} \div \frac{3}{7} = ?$

Change to like fractions.

$\frac{21}{28} \div \frac{12}{28} = 21 \div 12 = 1\frac{9}{12} = 1\frac{3}{4}$

Any number may be divided by a fraction by changing both numbers to like fractions and then dividing the numerator of the dividend by the numerator of the divisor.

8. Divide 6 by $\frac{3}{5}$

$6 = \frac{6}{1}$, for any whole number may be expressed as a fraction with 1 for its denominator.

$\frac{6}{1} \div \frac{3}{5}$ Change to like fractions.

$\frac{30}{5} \div \frac{3}{5} = 10$

9. Divide $\frac{3}{8}$ by 2.

$\frac{3}{8} \div \frac{2}{1}$ Change to like fractions.

$\frac{3}{8} \div \frac{16}{8} = 3 \div 16 = \frac{3}{16}$

By multiplying the dividend by the divisor inverted, the same result is obtained as in the process just described.

Thus,

$$\frac{3}{4} \div \frac{3}{7} = \frac{3}{4} \times \frac{7}{3} = \frac{7}{4} = 1\frac{3}{4}$$

$$6 \div \frac{3}{5} = \frac{6}{1} \times \frac{5}{3} = 10$$

$$\frac{3}{8} \div 2 = \frac{3}{8} \times \frac{1}{2} = \frac{3}{16}$$

To divide fractions, change to like fractions and divide the numerator of the dividend by the numerator of the divisor; or, for convenience, invert the divisor and multiply, canceling when possible.

Solve: *Written*

10. $\frac{3}{4} \div 6$	**11.** $\frac{7}{8} \div 14$	**12.** $\frac{9}{10} \div 15$	**13.** $\frac{5}{12} \div 10$
14. $\frac{7}{8} \div 21$	**15.** $\frac{8}{15} \div 12$	**16.** $\frac{5}{9} \div 5$	**17.** $\frac{8}{13} \div 16$
18. $2 \div \frac{3}{4}$	**19.** $10 \div \frac{4}{5}$	**20.** $15 \div \frac{3}{8}$	**21.** $18 \div \frac{6}{7}$
22. $16 \div \frac{8}{15}$	**23.** $14 \div \frac{7}{20}$	**24.** $9 \div \frac{6}{7}$	**25.** $14 \div \frac{7}{8}$
26. $\frac{14}{15} \div \frac{4}{5}$	**27.** $\frac{2}{3} \div \frac{4}{5}$	**28.** $\frac{7}{8} \div 1\frac{2}{13}$	**29.** $\frac{2}{9} \div \frac{4}{7}$
30. $\frac{3}{20} \div \frac{3}{8}$	**31.** $\frac{8}{20} \div \frac{9}{10}$	**32.** $\frac{4}{25} \div \frac{8}{15}$	**33.** $\frac{7}{18} \div \frac{4}{27}$
34. $\frac{3}{4} \div \frac{15}{16}$	**35.** $\frac{8}{9} \div \frac{1}{6}$	**36.** $\frac{7}{10} \div \frac{4}{5}$	**37.** $\frac{5}{12} \div \frac{9}{10}$

38. $2\frac{2}{3} \div 1\frac{1}{3}$

$$2\tfrac{2}{3} \div 1\tfrac{1}{3} = \frac{8}{3} \div \frac{4}{3} = \frac{\overset{2}{\cancel{8}}}{\underset{\cancel{3}}{\cancel{3}}} \times \frac{\cancel{3}}{\cancel{4}} = 2$$

39. $1\frac{2}{3} \div 10$	**40.** $2\frac{1}{2} \div \frac{4}{5}$	**41.** $\frac{3}{5} \div 1\frac{1}{2}$	**42.** $1\frac{3}{4} \div 2\frac{1}{3}$
43. $1\frac{1}{5} \div 18$	**44.** $1\frac{3}{8} \div \frac{3}{8}$	**45.** $\frac{4}{7} \div 1\frac{1}{4}$	**46.** $1\frac{1}{2} \div 1\frac{1}{4}$
47. $2\frac{1}{3} \div 21$	**48.** $2\frac{1}{7} \div \frac{1}{2}$	**49.** $\frac{3}{8} \div 1\frac{1}{3}$	**50.** $2\frac{1}{3} \div 3\frac{1}{4}$
51. $3\frac{5}{6} \div 2$	**52.** $3\frac{1}{2} \div \frac{7}{8}$	**53.** $\frac{7}{15} \div 2\frac{1}{2}$	**54.** $3\frac{1}{4} \div 2\frac{1}{5}$
55. $6\frac{3}{4} \div 5$	**56.** $2\frac{1}{3} \div \frac{7}{9}$	**57.** $\frac{4}{9} \div 3\frac{1}{4}$	**58.** $2\frac{2}{5} \div 1\frac{2}{3}$
59. $8\frac{1}{2} \div 3$	**60.** $4\frac{1}{5} \div \frac{14}{15}$	**61.** $\frac{5}{6} \div 1\frac{6}{7}$	**62.** $4\frac{1}{8} \div 3\frac{2}{3}$
63. $2\frac{5}{8} \div 7$	**64.** $5\frac{3}{8} \div \frac{7}{8}$	**65.** $\frac{4}{5} \div 4\frac{1}{5}$	**66.** $3\frac{2}{7} \div 5\frac{1}{2}$
67. $3\frac{3}{8} \div 9$	**68.** $1\frac{2}{3} \div 3\frac{1}{2}$	**69.** $\frac{3}{4} \div 2\frac{3}{4}$	**70.** $2\frac{4}{5} \div 1\frac{1}{4}$
71. $2\frac{5}{7} \div 4$	**72.** $3\frac{1}{2} \div \frac{2}{3}$	**73.** $\frac{7}{10} \div 3\frac{1}{3}$	**74.** $3\frac{2}{3} \div 1\frac{2}{3}$
75. $4\frac{2}{5} \div 11$	**76.** $4\frac{4}{5} \div \frac{4}{5}$	**77.** $\frac{9}{20} \div 12\frac{1}{2}$	**78.** $1\frac{7}{8} \div 3\frac{3}{4}$
79. $15\frac{3}{7} \div 4$			

(1) $4\overline{)15\frac{3}{7}}$ $\quad 3\frac{3}{7}$ — 4 in 15, 3 times, and 3 over. $3\frac{3}{7} = \frac{24}{7}$; $\frac{1}{4}$ of $\frac{24}{7} = \frac{6}{7}$.

(2) $15\frac{3}{7} \div 4 = \frac{\overset{27}{\cancel{108}}}{7} \times \frac{1}{\cancel{4}} = \frac{27}{7} = 3\frac{6}{7}$

80. $8\frac{2}{5} \div 3$	**81.** $9\frac{1}{7} \div 8$	**82.** $6\frac{9}{10} \div 4$	**83.** $7\frac{2}{3} \div 3$
84. $7\frac{1}{2} \div 6$	**85.** $7\frac{9}{10} \div 4$	**86.** $9\frac{4}{5} \div 7$	**87.** $7\frac{3}{16} \div 4$
88. $16\frac{5}{9} \div 7$	**89.** $12\frac{3}{4} \div 4$	**90.** $24\frac{3}{5} \div 9$	**91.** $15\frac{7}{8} \div 7$
92. $20\frac{5}{8} \div 9$	**93.** $12\frac{1}{2} \div 8$	**94.** $27\frac{3}{8} \div 6$	**95.** $17\frac{2}{7} \div 3$
96. $28\frac{3}{4} \div 5$	**97.** $25\frac{2}{5} \div 6$	**98.** $30\frac{5}{8} \div 3$	**99.** $50\frac{2}{3} \div 12$

PROBLEMS

Written

1. Mrs. Anderson paid \$$2\frac{1}{2}$ for braid at \$$\frac{1}{4}$ a yard. How many yards did she buy?

2. *How long* will it take to earn \$28 at \$$1\frac{3}{4}$ a day?

3. One and a half yards of cloth cost $\$1\frac{1}{5}$. What was he price of a yard?

4. How many books at $\$\frac{3}{4}$ apiece can be bought for $3\frac{3}{4}$?

5. How many boxes, each containing $\frac{5}{8}$ of a bushel, can ɔe filled from a barrel of apples ($2\frac{1}{2}$ bushels)?

6. If it takes $\frac{2}{3}$ of a yard of cretonne to cover a box, ıow many boxes can be covered with 10 yards?

7. A bill for shoeing a horse was $\$14$. If each shoeing cost $\$1\frac{3}{4}$, how many times was the horse shod?

8. At the rate of a mile in $2\frac{1}{2}$ minutes, it takes $\frac{1}{2}$ of an hour for a train to run from one station to another. How far apart are the stations?

9. A farmer exchanged 5 dozen eggs at $\$\frac{3}{10}$ a dozen 'or flour at $\$\frac{3}{4}$ a bag. How many bags of flour did he get?

10. It took 3 ten-yard rolls of ribbon for the diplomas f a graduating class. If $\frac{3}{4}$ of a yard was used for each ıploma, how many pupils were in the class?

FINDING WHAT PART ONE NUMBER IS OF ANOTHER *Oral*

1. What part of a dozen oranges is 1 orange? 6 ranges? 4? 8? 3? 9? 2? 10?

2. What part of a score (20) is 1? 10? 5? 15? 4? ? 12? 16? 2? 6? 14? 18?

3. What part of a score is 1 dozen? $\frac{1}{2}$ of a dozen? of a dozen? $\frac{2}{3}$ of a dozen? $\frac{1}{4}$ of a dozen? $\frac{3}{4}$ of a ozen? $\frac{1}{6}$ of a dozen? $\frac{5}{6}$ of a dozen?

4. Five girls made 24 pounds of candy for a country veek fund. On six days they sold 1 pound, 2 pounds,

3 pounds, 4 pounds, 6 pounds, 8 pounds. What part did they sell each day?

5. Edgar earned $20. He bought a hat for $1, a pair of shoes for $2, a reefer for $5, and a suit with the rest of his money. What part of his money did each article cost?

6. A contractor calls 48 hours a week's work. What part of a week's work is 12 hours? 36 hours? 16 hours? 32 hours? 8 hours? 40 hours? 6 hours? 18 hours? 30 hours? 42 hours? 24 hours? 48 hours?

PROBLEMS *Written*

1. In the month of September it rained 7 days and was cloudy 5 days. What part of the month was fair?

2. Lewis earns $6 a week and his father earns $20. Compare their wages.

3. Eugene puts in the bank 25 cents out of every dollar that he earns. What part of his money does he save? If his average earnings are a dollar a week, how much does he save in a year of 52 weeks?

4. A newsboy buys papers at 3 cents each and sells them at 5 cents each. How much does he gain? What part of the cost does he gain?

5. What part of the cost is gain when milk bought at 5 cents a quart is sold at 8 cents a quart?

6. Susan had 75 cents in her purse. She bought 6 handkerchiefs at the rate of 3 for a quarter. What part of her money did she spend?

7. Mr. Baxter bought a horse for $200. He sold it for $50 more than he paid. What part of the cost did *he gain?*

8. For collecting bills Howard received 10 cents out of every dollar collected. His pay was what part of the money collected?

9. At the rate of 10 cents on a dollar, what must be paid for collecting a bill of 40 dollars? At $12\frac{1}{2}$ cents?

10. What part of a barrel of kerosene containing 60 gallons is left after $\frac{3}{4}$ of it is sold to one customer and 5 gallons to another?

DRILL EXERCISE *Oral*

1. Find (1) $\frac{1}{2}$ of:

2	4	6	8	1	3	5	7	9

(2) $\frac{1}{4}$; $\frac{3}{4}$ (5) $\frac{1}{5}$; $\frac{2}{5}$; $\frac{3}{5}$; $\frac{4}{5}$ (8) $\frac{1}{9}$; $\frac{2}{9}$; $\frac{4}{9}$; $\frac{5}{9}$; $\frac{7}{9}$; $\frac{8}{9}$
(3) $\frac{1}{3}$; $\frac{2}{3}$ (6) $\frac{1}{8}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$ (9) $\frac{1}{7}$; $\frac{2}{7}$; $\frac{3}{7}$; $\frac{4}{7}$; $\frac{5}{7}$; $\frac{6}{7}$
(4) $\frac{1}{6}$; $\frac{5}{6}$ (7) $\frac{1}{12}$; $\frac{5}{12}$; $\frac{7}{12}$; $\frac{11}{12}$ (10) $\frac{1}{11}$; $\frac{2}{11}$; $\frac{3}{11}$; $\frac{5}{11}$; $\frac{7}{11}$; $\frac{10}{11}$

2. From each number in the row take each of the fractions.

3. Divide each number in the row by each of the fractions.

4. Make up simple problems based on the combinations in the above exercises.

FINDING THE WHOLE WHEN A PART IS GIVEN

Oral and Written

1. Nellie paid 15 cents for $\frac{3}{4}$ of a yard of cloth. What was the price per yard?

2. Ernest and Herbert bought a baseball. Ernest paid 20 cents. This was $\frac{2}{5}$ of the cost. How much did the ball cost? What part of the cost did Herbert pay? How much?

3. How long was the summer vacation if $\frac{2}{3}$ of it was 8 weeks?

4. Mrs. Wells paid 25 cents for thread. This was $\frac{5}{6}$ of what she paid for silk. How much did she pay for silk?

5. Ada is $\frac{2}{7}$ as old as her mother. Ada is 12 years old. How old is her mother?

6. A baseball team won 40 games. This was $\frac{5}{8}$ of the number played. How many games were played?

7. A load of wood cost 6 dollars. This was $\frac{3}{4}$ of the cost of a load of coal. How much did the coal cost?

8. After selling $\frac{3}{5}$ of a tub of butter, the grocer had 14 pounds left. How many pounds had he at first?

9. Ethel spelled 20 words correctly. She misspelled $\frac{1}{6}$ of the words given. How many words were given?

10. Mrs. Bradford bought $1\frac{1}{4}$ dozen, or $\frac{5}{4}$ dozen, buttons for 20 cents. What was the rate per dozen?

11. Roy has 24 marbles. This is $1\frac{1}{2}$, or $\frac{3}{2}$, times as many as Ralph has. How many has Ralph?

12. There are 35 boys in the fifth class. This is $1\frac{2}{5}$ times as many as in the sixth class. How many in the sixth class?

DRILL EXERCISE *Written*

In the following examples solve the part in parenthesis first.

Thus, $\frac{6}{7} \times (\frac{2}{3} + \frac{3}{4})$

$\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}$

$\frac{\overset{3}{\cancel{6}}}{7} \times \frac{17}{\underset{2}{\cancel{12}}} = \frac{17}{14} = 1\frac{3}{14}$

1. $\frac{3}{4} \times (\frac{1}{2} + \frac{4}{5})$
2. $(\frac{1}{2} + \frac{3}{4}) \times (\frac{1}{2} + \frac{2}{5})$
3. $1\frac{1}{4} \times (\frac{7}{8} - \frac{4}{5})$
4. $(\frac{5}{8} - \frac{2}{5}) \times (\frac{3}{4} + \frac{2}{3})$
5. $(1\frac{1}{2} + 1\frac{2}{3}) \times \frac{5}{6}$
6. $(\frac{3}{4} + \frac{1}{5}) \times (\frac{2}{3} - \frac{2}{5})$
7. $(3\frac{1}{4} + 2\frac{1}{7}) \times 6\frac{2}{7}$
8. $(\frac{3}{4} \times \frac{2}{3}) + (\frac{3}{5} + \frac{7}{8})$
9. $(1\frac{4}{5} - 1\frac{3}{15}) \times 1\frac{1}{5}$
10. $(\frac{2}{5} + \frac{7}{8}) - (\frac{3}{7} \times \frac{14}{15})$
11. $1\frac{4}{5} \times (1\frac{1}{5} - \frac{7}{15})$
12. $(1\frac{1}{2} + 2\frac{1}{5}) \times (\frac{2}{3} - \frac{1}{8})$
13. $(1\frac{1}{2} \times 4\frac{3}{8}) + 2\frac{2}{3}$
14. $(3\frac{2}{7} \times 2\frac{4}{5}) + (3\frac{1}{8} - 2\frac{1}{5})$
15. $(6\frac{7}{8} \times 1\frac{4}{5}) - 2\frac{7}{8}$
16. $(2\frac{3}{4} - 1\frac{2}{3}) \times (3\frac{2}{5} - 1\frac{3}{10})$
17. $(2\frac{1}{2} + 1\frac{5}{16}) \times 2\frac{2}{7}$
18. $(\frac{3}{8} + \frac{7}{8}) - (\frac{3}{4}$ of $\frac{8}{15})$
19. $(\frac{2}{3} + \frac{2}{3}) \times 2\frac{1}{4}$
20. $(\frac{5}{6}$ of $\frac{2}{3}) \times (\frac{7}{8} + \frac{2}{3})$

REVIEW PROBLEMS *Written*

1. What must be paid for 168 bushels of wheat at $\$\frac{4}{5}$ a bushel?

2. Two tubs of maple sugar weighed $42\frac{5}{16}$ pounds. One weighed $18\frac{3}{8}$ pounds. What did the other weigh?

3. How many dozen eggs at $\frac{1}{5}$ of a dollar a dozen can be bought for $2\frac{1}{2}$ dollars?

4. A shoe dealer paid $\$2\frac{4}{5}$ for a pair of shoes. He sold them at a gain of $\$\frac{1}{2}$. For how much did he sell them?

5. If a yard of cloth costs $\$\frac{4}{5}$, what part of a dollar will $\frac{1}{2}$ of a yard cost? $\frac{1}{4}$ of a yard? $\frac{3}{4}$ of a yard? $1\frac{3}{4}$ yards?

6. James bought a pair of rabbits for $\$1\frac{7}{20}$ and sold them at a loss of $\$\frac{1}{4}$. For how much did he sell them?

7. Sidney buys newspapers at $1\frac{1}{2}$ cents each and sells them for 2 cents each. How many must he sell to earn one dollar?

8. Into how many pieces $\frac{2}{3}$ of a foot long can a stick $7\frac{1}{3}$ feet long be cut?

9. What is the cost of $2\frac{1}{2}$ yards of cloth at $\frac{3}{5}$ of a dollar a yard and $1\frac{1}{2}$ yards at $1\frac{1}{2}$ dollars a yard?

10. Mrs. Sherman owed her grocer $\$1\frac{1}{2}$, $\$3\frac{2}{5}$, $\$2\frac{4}{5}$, and $\$1\frac{7}{10}$. She gave a \$10-bill in payment. How much did the grocer return?

11. If a ticket to Denver costs $\$\frac{3}{5}$, how much ought a ticket to cost to a place $\frac{5}{6}$ as far away? To a place 3 times as far away? To a place $1\frac{1}{4}$ times as far away?

12. A stationer gained $\$\frac{1}{5}$ by selling a book for $\$1\frac{1}{4}$. Find the cost.

13. The rent of a house for 15 weeks was $\$41\frac{1}{4}$. What was the rent for 1 week?

14. At $\$1\frac{1}{5}$ per rod, what will $10\frac{1}{2}$ rods of fence cost?

15. The deposits made in a bank were $84\frac{3}{4}$ dollars, $62\frac{1}{2}$ dollars, $75\frac{3}{5}$ dollars. The withdrawals were $15\frac{6}{25}$ dollars, $9\frac{7}{10}$ dollars, and $67\frac{1}{4}$ dollars. How much was left in the bank?

16. How many sacks of potatoes at $\$1\frac{1}{4}$ a sack will pay for 15 yards of cloth at $\$\frac{1}{2}$ a yard?

17. A man owning $\frac{1}{4}$ of a mill sold $\frac{1}{2}$ of his share for \$5000. At that rate what was the whole mill worth?

18. If $\frac{7}{8}$ of a quart fills a bottle, how many bottles can be filled from $17\frac{1}{2}$ quarts of maple sirup?

19. An express train runs $\frac{3}{4}$ of a mile in $\frac{7}{8}$ of a minute. What is the rate a minute? How long does it take to run a mile?

20. A gang of men laid $\frac{1}{8}$ of a sewer in one day, $\frac{3}{10}$ the next day, and $\frac{1}{5}$ the third day. What part of the work *was then done?* What part remained to be done?

21. After selling $\frac{3}{4}$ of a barrel of oil, the grocer had 9 gallons left. How many gallons had he at first?

22. What is the sum of the ages of 5 children whose ages are, respectively, $6\frac{5}{12}$ years, $8\frac{7}{12}$ years, $10\frac{3}{4}$ years, $7\frac{1}{2}$ years, and $9\frac{1}{4}$ years? What is the average age?

23. A farmer can mow a piece of grain in 4 hours. His son can mow it in 6 hours. If both work together, what part of the piece can they mow in 1 hour?

24. A gardener planted 80 geraniums. 24 of them died. What part lived?

25. A plumber is paid $\$4\frac{1}{2}$ a day, and his helper $\frac{2}{5}$ as much. How much is paid to both?

DECIMALS

A *power* of a number is the product obtained by using that number as a factor a specified number of times. Thus, the second power of 10 is 10×10, or 100; the third power of 10 is $10 \times 10 \times 10$, or 1000; and so on.

Fractions whose denominators are 10, 100, 1000, etc., as $\frac{7}{10}$, $\frac{7}{100}$, $\frac{7}{1000}$, that is, common fractions whose denominators are 10, or some power of 10, are *decimal fractions.*

When the denominators are expressed by a decimal point, as in .7, .07, .007, such fractions are called *decimals.*

The word "decimal" comes from the Latin word *decem,* which means ten.

When a decimal fraction is expressed as a decimal, there are as many places in the decimal as there are 0's in the denominator of the decimal fraction. Thus, $\frac{9}{10} = .9$; $\frac{9}{100} = .09$; $\frac{9}{1000} = .009$; $\frac{9}{10000} = .0009$; and so on.

When a decimal is expressed as a common fraction, the denominator is 1 with as many ciphers annexed as there are places in the decimal. Thus, $.7 = \frac{7}{10}$; $.07 = \frac{7}{100}$; $.007 = \frac{7}{1000}$; $.0007 = \frac{7}{10000}$; and so on.

Write as decimals: $\frac{3}{10}$ $\frac{19}{100}$ $\frac{123}{1000}$

Write as common fractions: .9 .13 .217

Always treat a decimal as if it were a whole number.

The decimal point simply tells the kind of things with which we are dealing; that is, it gives a name to the decimal.

READING AND WRITING DECIMALS

The following table shows the method of reading and writing decimals.

NAMES OF PLACES	Decimal point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
ORDER OF PLACES	●	1st	2d	3d	4th	5th	6th

Names and order of places must be memorized.

A decimal takes its name from its denominator.

1. Read .0425.

Read the numerator as you would any whole number—four hundred twenty-five. 425 what? The place of the last figure in the numerator shows the name of the denominator. The last figure, 5, stands in the fourth, or ten-thousandths' place. Therefore, we read four hundred twenty-five ten-thousandths.

2. Read .000007.

Seven what? The figure 7 stands in the sixth or millionths' place. We read, seven millionths.

3. Read: .1; .01; .001; .0001; .00001; .000001.

4. .672	5. .5463	6. .34927	7. .372624
8. .308	9. .4039	10. .40671	11. .607503
12. .056	13. .5708	14. .30058	15. .800465
16. .009	17. .0432	18. .05046	19. .070324
20. .666	21. .0704	22. .00302	23. .030012
24. .404	25. .0078	26. .07005	27. .007009
28. .088	29. .6005	30. .00049	31. .000605
32. .005	33. .0004	34. .00008	35. .000003

36. Read 27.56.

The whole number is 27; the decimal is 56 hundredths. Using the word "and" to mark the decimal point, we read, twenty-seven and fifty-six hundredths.

In order to avoid confusion in reading numbers, use the word "and" only to mark the separation between a whole number and a decimal. Thus, 106.07 is read one hundred six *and* 7 hundredths.

Numbers consisting of a whole number and a decimal are *mixed decimals*.

Read these mixed decimals:

37. 2.2 20.2 202.02 202.202 200.002

38. 243.6	39. 375.64	40. 384.018	41. 300.003
42. 35.72	43. 23.071	44. 58.6437	45. 4000.04
46. 3.628	47. 8.0706	48. 547.009	49. 50.0005
50. 58.07	51. 93.007	52. 39.0003	53. 60000.6
54. 80.09	55. 70.024	56. 8.00005	57. 7.00007

58. How many decimal places are required to express tenths? To express hundredths? Thousandths? Ten-thousandths? Hundred-thousandths? Millionths?

59. Write the figure 6 so that it shall express tenths; hundredths; thousandths; ten-thousandths; hundred-thousandths; millionths.

60. Write in figures six hundred eight ten-thousandths.

Ten-thousandths shows that the last figure of the decimal stands in the fourth decimal place, so we write .0608.

61. Write in figures six hundred and eight ten-thousandths. The whole number is 600, the decimal .0008; so we write 600.0008.

Write in figures:

62. Seven and three hundredths.

63. Nine thousandths.

64. Two hundred eighty-six ten-thousandths.

65. Two hundred and eighty-six ten-thousandths.

66. Seven hundred fifty-two hundred-thousandths.

67. Thirty-six and eight hundred-thousandths.

68. Five hundred twenty-seven millionths.

69. Nine and three thousand forty-six millionths.

70. Seventeen and one hundred three ten-thousandths.

71. Sixty and sixteen ten-thousandths.

72. Four and four thousandths.

73. Four hundred and four thousandths.

74. *Four* hundred four thousandths.

CHANGING DECIMALS TO COMMON FRACTIONS *Oral*

1. What is the denominator in .2? .7? .12? .08? .125? .015? .004?

2. Change .08 to a common fraction in its lowest terms.

$$.08 = \tfrac{8}{100} = \tfrac{2}{25}$$

3. Change .075 to a common fraction.

$$.075 = \tfrac{75}{1000} = \tfrac{3}{40}$$

4. Change $.33\frac{1}{3}$ to a common fraction.

(1) $$.33\tfrac{1}{3} = \frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{100}{3} \times \frac{1}{100} = \frac{1}{3}$$

(2) $$.33\tfrac{1}{3} = \frac{33\frac{1}{3} \times 3}{100 \times 3} = \frac{100}{300} = \frac{1}{3}$$

To change a decimal to a common fraction, write the numerator over the denominator expressed in figures, and change to lowest terms.

Such expressions as .6, .375, .006, that is, decimals in which the numerator is a whole number, are *simple* or *pure decimals.*

Such expressions as $.33\frac{1}{3}$, $.37\frac{1}{2}$, $.14\frac{2}{7}$, that is, decimals in whose numerator there is a common fraction, are *complex decimals.*

Name the common fractions equivalent to:

5. .1 .2 .3 .4 .5 .6 .7 .8 .9

6. .10 .20 .30 .40 .50 .60 .70 .80 .90

Written

Change to common fractions in their lowest terms:

7. .02 8. .04 9. .06 10. .08

11. .12 12. .16 13. .24 14. .32

15. .36	16. .48	17. .64	18. .72
19. .25	20. .75	21. .35	22. .85
23. .125	24. .375	25. .625	26. .875
27. .12½	28. .37½	29. .62½	30. .87½
31. .33⅓	32. .66⅔	33. .16⅔	34. .83⅓

CHANGING COMMON FRACTIONS TO DECIMALS *Oral*

1. Change $\frac{2}{25}$ to a decimal.

$$\frac{2}{25} \times \frac{4}{4} = \frac{8}{100} = .08$$

Since the denominator of a decimal is 10 or some power of 10, we must change $\frac{2}{25}$ to a fraction whose denominator is 10, 100, 1000, etc. Multiplying both terms of $\frac{2}{25}$ by 4, we get $\frac{8}{100}$. Writing $\frac{8}{100}$ in decimal form, we get .08.

Give the decimal equivalent of:

2. $\frac{1}{2}$	3. $\frac{1}{4}$	4. $\frac{3}{4}$	5. $\frac{1}{5}$	6. $\frac{2}{5}$
7. $\frac{3}{5}$	8. $\frac{4}{5}$	9. $\frac{1}{10}$	10. $\frac{3}{10}$	11. $\frac{7}{10}$
12. $\frac{9}{10}$	13. $\frac{1}{20}$	14. $\frac{11}{20}$	15. $\frac{1}{25}$	16. $\frac{4}{25}$
17. $\frac{13}{25}$	18. $\frac{1}{50}$	19. $\frac{3}{50}$	20. $\frac{17}{50}$	21. $\frac{42}{50}$

22. Compare: \$1 \$1. \$1.0 \$1.00 \$1.000

23. Compare: 5 5.0 5.00 5.000

24. Is this statement true?

$$8)\underline{5} = 8)\underline{5.} = 8)\underline{5.0} = 8)\underline{5.00} = 8)\underline{5.000}$$

When the decimal equivalent is not readily seen, change a common fraction to a decimal by dividing the numerator by the denominator.

25. Change $\frac{3}{5}$ to a decimal.

$$5)\underline{3.0}$$
$$.6$$

26. Change $\frac{5}{8}$ to a decimal.

$$8)\underline{5.000}$$
$$.625$$

In both these examples we place a decimal point after the numerator, and annex as many 0's as may be necessary.

To change a common fraction to a decimal, divide the numerator by the denominator.

Change to decimals: *Written*

27. $\frac{1}{8}$ 28. $\frac{3}{8}$ 29. $\frac{7}{8}$ 30. $\frac{1}{16}$ 31. $\frac{3}{16}$

32. $\frac{5}{16}$ 33. $\frac{1}{32}$ 34. $\frac{1}{40}$ 35. $\frac{3}{40}$ 36. $\frac{1}{80}$

Since the denominator of a decimal is always 10 or some power of 10, the only common fractions that can be expressed as pure decimals are those whose denominators, when the fractions are in their lowest terms, contain only the factors 2 or 5.

37. Which of these fractions can be expressed as pure decimals?

$\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{5}{6}$ $\frac{6}{7}$ $\frac{7}{8}$ $\frac{8}{9}$ $\frac{9}{10}$ $\frac{5}{12}$

$\frac{3}{14}$ $\frac{2}{15}$ $\frac{7}{16}$ $\frac{5}{18}$ $\frac{9}{20}$ $\frac{11}{24}$ $\frac{2}{25}$ $\frac{7}{30}$ $\frac{13}{40}$ $\frac{3}{50}$

38. Change $\frac{1}{3}$ to a decimal.

$\frac{1}{3} = 3\overline{)1.0}$, or $3\overline{)1.00}$, or $3\overline{)1.000}$, and so on.

$3\overline{)1.0} = 3\overline{)1.0}$, or $3\overline{)1.0}$
$.3\frac{1}{3}$ $.3+$

$3\overline{)1.00} = 3\overline{)1.00}$, or $3\overline{)1.00}$
$.33\frac{1}{3}$ $.33+$

$3\overline{)1.000} = 3\overline{)1.000}$, or $3\overline{)1.000}$
$.333\frac{1}{3}$ $.333+$

That is, there will always be a remainder. Show this either by writing the undivided remainder as a fraction, or by writing a + sign after the last quotient figure.

Change to decimals carrying out the division three decimal places:

39. $\frac{2}{3}$	40. $\frac{1}{6}$	41. $\frac{5}{6}$	42. $\frac{1}{7}$	43. $\frac{3}{7}$
44. $\frac{5}{7}$	45. $\frac{1}{9}$	46. $\frac{4}{9}$	47. $\frac{8}{9}$	48. $\frac{1}{12}$
49. $\frac{5}{12}$	50. $\frac{7}{12}$	51. $\frac{1}{15}$	52. $\frac{8}{15}$	53. $\frac{11}{15}$

DECIMALS: ADDITION

Oral

Find the sum of:

1. .5 .5	2. .9 .4	3. 1.3 .7	4. 2.4 .8	5. 2.5 1.5
6. .5 .25	7. .25 .75	8. 1.5 2.5	9. 2.25 3.25	10. 1.58 .3

11. Mrs. Marston paid .3 of a dollar for sugar and .5 of a dollar for tea. What part of a dollar did she pay for both?

12. A family used .25 of a ton of coal in one month and .5 of a ton the next month. How much in both months?

13. Two tenths of a garden is given to roses, .3 to dahlias, and .25 to asters. What part of the garden is given to flowers?

14. Stella is 8.5 years old and Helen is 7.5 years old. What is the sum of their ages?

15. In canning berries, Mrs. Libby used 7.5 pounds of sugar and .25 of a pound of spice. What did both weigh?

WRITTEN EXERCISE

1. Add .5, .04, .785, .03, .018, and .07.

.5
.04
.785
.03
.018
.07

In arranging decimals in columns, that is, in writing them so that tenths are in one column, hundredths in another, and so on, we are really changing them to a common denominator.

Find the sum of:

2. .8, .64, .73, .3
3. 1.2, 3.7, .5, 1.9
4. 3.375, 5.06, 9.003, 7.064
5. 160.5, 128.7, 64.03, 98.01
6. 37.55, 64.98, 121.06, 112.003
7. .003, .0104, .08005, .0004
8. 65, 9.008, 13.05, 12.004
9. 100.56, 84.07, 93.005, 86.002
10. 1012.5, 648.03, 954.002, 671.08
11. .6, .06, .006, .0006, .00006

12. 4 tenths; 75 thousandths; 3 hundredths; 12 ten-thousandths; 65 thousandths.

13. Sixty-four ten-thousandths; one hundred four thousandths; three tenths; eight ten-thousandths.

14. One hundred ten thousandths; twenty-six thousandths; six hundred forty-seven thousandths; eighty-five hundred-thousandths.

15. Two hundred ten thousandths; two ten-thousandths; two hundred ten thousand.

16. Seven hundred and seven tenths; one hundred four and sixty-five thousandths; seventy-five and five hundredths; forty and sixty-five ten-thousandths.

PROBLEMS *Written*

1. How many tons of coal in three loads weighing, respectively, 1.75 tons, 1.875 tons, and 2.5 tons?

2. The four sides of a lot measure 20.8 yards, 27.5 yards, 26.25 yards, and 21.75 yards. What length of fence will inclose it?

3. A bicycle cyclometer registered the following distances on five successive days: 12.9 miles, 17.25 miles, 14.8 miles, 20.65 miles, and 16.5 miles. What was the entire distance traveled?

4. A farmer purchased five adjoining fields whose areas were 3.675 acres, .85 of an acre, 1.005 acres, 4.012 acres, and 7.2 acres. How many acres in all?

5. Mr. Simpson drove his automobile 46.2 miles, 38.5 miles, and 40.75 miles. His speedometer read 130.3 miles at the start. What did it read at the finish?

DECIMALS: SUBTRACTION *Oral*

Find differences:

1. 1.8	2. 1.2	3. .50	4. 8.3	5. .07
.8	.5	.25	2.1	.02
6. 5.75	7. 2.75	8. 2.50	9. 2.3	10. 2.2
.5	.25	.25	.5	1.5

11. From 1 take .1; .2; .8; .9.

12. From 1 take .5; .25; .75.

13. From 2 take 1.5; 1.25; 1.75.

14. When you have completed .6 of your journey, what part remains to be traveled?

15. If you spend .85 of your money, what part have you left?

16. Mrs. Joslin bought a yard of silk and used .75 of it for trimming. What part of a yard had she left?

17. Four tenths of a bushel of apples decayed. What part was good?

18. Dora was given .75 of a dollar. She spent .2 of a dollar for a belt and .1 of a dollar for ice cream. What part of a dollar had she then?

WRITTEN EXERCISE

1. From .6 take .35; .035; .0035.

2. From .08 take .04; .004; .0004.

3. From .004 take .003; .0003.

4. From 6.75 take .305.

5. Take .03 from 30.1.

6. From .04 take .018.

7. Take 7.2 from 26.01.

8. From .9 take .36.

9. Take 3.05 from 10.7.

10. From 8.06 take 6.08.

11. Take .015 from 6.4.

12. From .764 take .0647.

13. Take .0085 from 3.7.

14. Subtract two tenths from two.

15. What is the difference between ten and one tenth, and one and one tenth?

16. What is left when six and seven hundredths is taken from eight and twenty-four thousandths?

17. Find the difference between seventy-six and five hundredths, and fifty-nine and eight thousandths.

18. From five take seven thousandths.

19. Take thirteen hundredths from two hundred one thousandths.

20. From one and one hundred sixty-two ten-thousandths take one hundred twenty-two thousandths.

21. From three and twenty-five thousandths take forty-nine hundredths.

PROBLEMS *Written*

1. A owns .42 of a vessel, and B the remainder. What part does B own?

2. What remains, if from .8 of a bushel of corn .375 of a bushel is taken?

3. If Harvey loses .625 of his kite string, what part has he left?

4. If Joseph spends .125 of his money for a ball, .4 for a knife, and .25 for a bat, what part of his money has he left?

5. A French franc piece is worth nineteen and three tenths cents. How much less than one fifth of a United States dollar?

6. Which is the greater quantity, $\frac{3}{4}$ of a pound of butter or .875 of a pound of butter? How much greater?

7. A man left .5 of his property to his wife, .225 to his son, and the rest to his daughter. What was the daughter's share?

8. A man does a piece of work in five days. What part of the work does he do the last day, if the work of the first four days is as follows: .25, .15, .165, .2?

9. A received $\frac{1}{2}$ of an estate, and B .45 of the estate. *Which* received the greater share? By how much?

DECIMALS: MULTIPLICATION *Oral*

Give products:

1. .5	**2.** 12	**3.** 8	**4.** .7	**5.** .3
2	.5	.4	5	2
6. .5	**7.** .25	**8.** .75	**9.** 2.2	**10.** 1.25
6	4	2	6	5

11. Multiply by 3: .2 .02 .002

12. Multiply by 2: .8 .06 .007

13. Multiply by 5: .4 .08 .006

14. Multiply by 4: .5 .05 .005

15. A newsboy sold .9 of his 50 papers. How many?

16. Twenty-five hundredths of a flock of 80 sheep were sold. How many?

17. A suit cost $15 and a hat .2 as much. What did the hat cost?

18. How much is .4 of $600?

19. When Mr. Spaulding has used .75 of his 12 tons of coal, how many tons remain?

20. Multiply .7 by .3.

$\frac{7}{10} \times \frac{3}{10} = \frac{21}{100} = .21$

.7
.3
.21

Change the decimals to common fractions and multiply. $\frac{7}{10} \times \frac{3}{10} = \frac{21}{100}$. Expressing $\frac{21}{100}$ in decimal form, we get .21. In multiplying .7 by .3, it is evident that, since the denominators are 10 and 10, the denominator of the product must be 10 × 10, or 100. A decimal whose denominator is 100 occupies two decimal places, which is the sum of the decimal places in the multiplicand (.7) and the multiplier (.3).

21. Multiply .07 by .3.

$\frac{7}{100} \times \frac{3}{10} = \frac{21}{1000} = .021$

.07
.3
.021

The product of the denominators is 1000. A decimal whose denominator is 1000 occupies three decimal places, which is the sum of the decimal places in the multiplicand (.07) and the multiplier (.3).

To multiply decimals, multiply as in whole numbers and point off in the product as many decimal places as there are decimal places in both multiplicand and multiplier.

Note that the "pointing off" is simply the multiplying together of the denominators.

22. Without multiplying, tell how many decimal places in the product of:

.8 × 4	.03 × .6	.004 × .2	.005 × .07
7 × .6	.4 × .09	.6 × .004	.03 × .009

Written

Solve:

23. .4 × 12	**24.** 8 × .15	**25.** 1.08 × 5
26. .64 × .25	**27.** 6.25 × 14	**28.** .35 × .12
29. 15.6 × 2.2	**30.** 8.02 × 1.06	**31.** 8.8 × .08
32. 3.4 × 2.5	**33.** 3.6 × .005	**34.** .375 × .44
35. 37.2 × 1.05	**36.** .075 × 24	**37.** 1.8 × 2.25
38. 8.07 × 1.6	**39.** 62.5 × .32	**40.** 60.2 × 4.9
41. .44 × .265	**42.** .015 × 144	**43.** 87.5 × .06
44. 4.26 × .62	**45.** 3.125 × .8	**46.** 124 × .75
47. .025 × 1.24	**48.** 6.6 × .031	**49.** 12.8 × 4.2
50. *123 × .375*	**51.** 4.8 × .085	**52.** 16.5. × 4.82

MULTIPLYING BY 10, 100, 1000 *Oral*

1. Move the decimal point in $2.000 one place to the right. What number have you now? How does $20.00 compare with $2.000? By what have you multiplied $2.000?

2. Move the decimal point in $2.000 two places to the right. What have you done to $2.000?

3. What is the result of moving the decimal point in $2.000 three places to the right? What have you done to $2.000?

4. Multiply (1) by 10; (2) by 100; (3) by 1000:

256 34.7 7.28 .175 .0306 .0008

5. One lot contained .755 of an acre, and another lot 10 times as much. What was the area of the second lot?

6. There are 272.25 square feet in a square rod. How many in 100 square rods?

7. At $2.75 a yard, how much will a merchant pay for 1000 yards of silk?

MULTIPLYING BY .1, .01, .001

Multiply 444.4 by .1; by .01; by .001.

To multiply a number by .1 is to find one tenth of it.

1. In multiplying by .1, how many places, and in what direction, is the decimal point moved?

2. Tell how to multiply by .01. By .001.

3. Multiply (1) by .1; (2) by .01; (3) by .001:

1247 300 1.2 4.03 .542 .08

Solve:

4. 10×24.6	**5.** $1000 \times .35$	**6.** $100 \times .08$
7. $.01 \times 57.2$	**8.** $.001 \times 16.07$	**9.** $.1 \times 3.75$
10. $.1 \times .004$	**11.** $.01 \times .012$	**12.** 10×1.05
13. $100 \times .016$	**14.** 1000×1.035	**15.** $.01 \times .0007$
16. $.001 \times 1.44$	**17.** 10×5.27	**18.** $.001 \times 30.75$

19. Find by groups:

.1 of 5280 ft.	320 rd.	16.5 ft.	5.5 ft.	3 ft.	12 in.
.01 of 5280 ft.	320 rd.	16.5 ft.	5.5 ft.	3 ft.	12 in.
.001 of 5280 ft.	320 rd.	16.5 ft.	5.5 ft.	3 ft.	12 in.

PROBLEMS *Written*

1. Forty-five hundredths of the 160 acres in Mr. Cheever's farm is under cultivation. How many acres?

2. At .64 of a dollar a bushel, how much will 12.5 bushels of oats cost?

3. A rod is 16.5 feet. How many feet in 4.8 rods?

4. There are 5280 feet in a mile. How many feet in .375 of a mile?

5. How much will .75 of a bushel of apples cost at .8 of a dollar a bushel?

6. At $\$0.08\frac{3}{4}$ a foot, what will it cost to fence a four-sided field, 120 feet on a side?

7. At different times a man bought .8 of an acre of land, .375 of an acre, 2.25 acres, and 1.5 acres. At $54 per acre, how much did it all cost?

8. A farmer has two fields, one containing 12.75 acres and the other .6 as many acres. How many acres in the second field? How many acres in both fields?

9. Mr. Bean's farm contains 240 acres. If he raises hay on .12½ of it, and harvests 1.5 tons to the acre, how many tons does he get?

10. Two men each owned .8 of an acre of land. One sold .375 of an acre. What part of an acre had he left? The other sold .375 of his lot. What part of an acre had he left?

DECIMALS: DIVISION *Written*

Test these divisions:

1.	2.	3.	4.	5.
5)$0.35	5)$0.035	3).6	3).06	3).006
$0.07	$0.007	.2	.02	.002

In dividing a decimal by a whole number, the decimal point in the quotient stands in the same column as the decimal point in the dividend.

The first step in division of decimals is to write the decimal point in the quotient.

Divide, and test:

6. 2).8	7. 2).24	8. 2).024	9. 2).008
10. 3).9	11. 3).15	12. 3).027	13. 3).009
14. 6).6	15. 6).36	16. 6).036	17. 6).006
18. 5).5	19. 5).15	20. 5).015	21. 5).005

22. Divide .1 by 5. 5).1 = 5).10

Divide, and test:

23. 5).2	24. 5).02	25. 5).002	26. 5).012
27. 4).2	28. 4).02	29. 4).002	30. 4).018
31. 6).3	32. 6).03	33. 6).003	34. 6).015
35. 8).4	36. 8).04	37. 8).004	38. 8).012
39. 8).1	40. 4).03	41. 4).003	42. 4).034

43. .012 by 3	**44.** .56 by 7	**45.** .92 by 8
46. .84 by 6	**47.** .3675 by 5	**48.** .873 by 9
49. .1272 by 12	**50.** .728 by 7	**51.** .091 by 13
52. .016 by 5	**53.** .315 by 15	**54.** .64 by 16
55. .28 by 35	**56.** .0108 by 12	**57.** .3311 by 11
58. 9.31 by 7	**59.** 17.28 by 12	**60.** 2.75 by 25
61. 4.32 by 36	**62.** 42.84 by 42	**63.** 17.1 by 57
64. 771.2 by 64	**65.** 1.98 by 18	**66.** 12.376 by 34
67. 10.08 by 72	**68.** 19.95 by 19	**69.** 1.552 by 97
70. 97.44 by 40	**71.** 438.9 by 21	**72.** .0774 by 86
73. 6.6 by 75	**74.** 378.2 by 61	**75.** 1.394 by 17

76. Divide .24 by .3 $\quad .3\overline{).24} = 3\overline{)2.4}$

When there is a decimal in the divisor, change the divisor to a whole number by multiplying both divisor and dividend by 10, 100, 1000, etc. Thus, change $.3\overline{).24}$ to $3\overline{)2.4}$ by multiplying the divisor .3 by 10, and the dividend .24 by 10.

What is done in each of these cases to make the divisor a whole number?

77. $.2\overline{)1.6} = 2\overline{)16}$	**78.** $.03\overline{).015} = 3\overline{)1.5}$
79. $.06\overline{).42} = 6\overline{)42}$	**80.** $.7\overline{)28} = 7\overline{)280}$
81. $.04\overline{)1.28} = 4\overline{)128}$	**82.** $.8\overline{).024} = 8\overline{).24}$
83. $.02\overline{).0012} = 2\overline{).12}$	**84.** $.005\overline{).35} = 5\overline{)350}$

Divide, first making the divisor a whole number:

85. $.7\overline{)8.4}$	**86.** $.9\overline{).081}$	**87.** $.04\overline{).036}$
88. $.8\overline{).56}$	**89.** $.2\overline{).72}$	**90.** $.2\overline{)7.2}$
91. $.6\overline{)72}$	**92.** $.2\overline{).072}$	**93.** $.08\overline{)1.08}$

94. .07)$\overline{.0014}$	**95.** .06)$\overline{16.2}$	**96.** .09)$\overline{135}$
97. .9)$\overline{.063}$	**98.** .009)$\overline{.054}$	**99.** .09)$\overline{81.}$
100. .09)$\overline{10.8}$	**101.** 1.2)$\overline{1.4}$	**102.** .12)$\overline{14.4}$
103. .12)$\overline{.0144}$	**104.** .012)$\overline{1.44}$	**105.** .18)$\overline{17.28}$
106. .27)$\overline{1.728}$	**107.** 3.6)$\overline{17.28}$	**108.** .054)$\overline{.1728}$

Solve:

109. .625 ÷ 25	**110.** 9.36 ÷ .6	**111.** 17.28 ÷ .12
112. 64.68 ÷ 8.8	**113.** 445.2 ÷ .42	**114.** 12.72 ÷ .08
115. .66 ÷ 120	**116.** 60 ÷ .625	**117.** .012 ÷ .15
118. 300 ÷ .75	**119.** 19.68 ÷ .802	**120.** 14.4 ÷ .006
121. 768.8 ÷ .31	**122.** .0012 ÷ .24	**123.** 8.04 ÷ 1.05
124. 24.98 ÷ .1	**125.** 37.64 ÷ .01	**126.** 5.32 ÷ .01
127. .693 ÷ 4.5	**128.** 15.912 ÷ .52	**129.** 465 ÷ 3.75
130. 513.9 ÷ 4	**131.** 725 ÷ .025	**132.** .2727 ÷ 3.03
133. 315 ÷ 2.52	**134.** .2544 ÷ .01	**135.** 2400 ÷ 9.6

PROBLEMS *Written*

1. A field containing 7.2 acres was cut up into lots of .4 of an acre each. How many lots?

2. In 6 days a grocer sold 21.6 tubs of butter. What was the average quantity sold a day?

3. A pile of books is 33.75 inches high. If each book is .75 of an inch thick, how many books in the pile?

4. The distance between two places is 15.75 miles. How long will it take a railroad train running at the rate of .45 of a mile a minute to make the trip?

5. How many barrels of 2.5 bushels each must Mr. Davis buy to barrel 80 bushels of apples?

6. What is the average speed per hour of an automobile which goes 117.81 miles in 6.3 hours?

7. One hundred sixty-five bushels of potatoes were put up in bags of 1.5 bushels each. How many bags were used?

8. A bushel of grain occupies 1.25 cubic feet. How many bushels can be put into a bin whose capacity is 17.5 cubic feet?

9. A wheel of a wagon is 16.5 feet in circumference. How many revolutions will it make in going 1 mile (5280 ft.)?

10. An automobile speedometer registered 184.6 miles at the beginning of a trip, and 324.1 miles at the end. The trip was made in 6.2 hours. What was the distance traveled per hour?

DIVIDING BY 10, 100, 1000 *Oral*

$333 \div 10 = 33.3$
$333 \div 100 = 3.33$
$333 \div 1000 = .333$

1. Tell how to divide a number by 10. By 100. By 1000.

Solve:

2. $10 \div 10$	3. $2.5 \div 10$	4. $76.5 \div 100$
5. $37.5 \div 1000$	6. $273 \div 100$	7. $.002 \div 10$
8. $24.64 \div 10$	9. $10.01 \div 10$	10. $4.03 \div 10$
11. $24 \div 100$	12. $374.2 \div 1000$	13. $.5 \div 10$
14. $12.5 \div 10$	15. $.4 \div 100$	16. $3.7 \div 100$

17. Ten acres of land were sold for $225. What was the rate per acre?

18. A railroad company carried 1000 people on an excursion for $750. How much did each person pay?

19. What was the cost of a ton of coal if 100 tons cost $565?

REVIEW EXERCISE *Written*

1. Write in figures, five hundred five thousandths.

2. Write in figures, five hundred and five thousandths.

3. Write in words, 142.025.

4. Write in words, .0106.

5. What is the denominator in .8? .036? .04? .3786?

6. How many decimal places in $\frac{9}{10}$ when written as a decimal? $\frac{3}{100}$? $\frac{741}{1000}$? $\frac{7}{1000}$?

7. Express as decimals: $\frac{3}{5}$ $\frac{3}{10}$ $\frac{3}{20}$ $\frac{3}{25}$ $\frac{3}{50}$

8. Express as common fractions in their lowest terms: .5 .50 .70 .06 .85 $.1\frac{1}{2}$ $.2\frac{1}{2}$

9. Find decimally the sum of $\frac{1}{4}$ and $\frac{1}{8}$.

10. What is the figure in the third decimal place when $\frac{1}{7}$ is expressed as a decimal?

11. What is the quotient of $4 \div .4$? Of $.4 \div 4$?

12. I have $5. What is .2 of it? .5?

13. $5 is .2 of my money. How much have I?

14. If 4 dozen oranges cost .8 of a dollar, what will 12 dozen cost?

15. If a peck of potatoes costs .4 of a dollar, what will a bushel cost?

16. Three caps cost .6 of a dollar. How many can be bought for a dollar?

17. $.2 \times .3 \times 1 = ?$

18. $.2 \times .3 \times 1 \times 0 = ?$

19. If you receive .05 of what you collect, how much will you receive for collecting $32?

20. What part of 27 is 9? Express the answer decimally.

21. Howard and Fred each earned .8 of a dollar. Howard spent .5 of a dollar at the circus. How much had he left? Fred spent .5 of his money in car rides. How much had he left?

Solve by groups:

22. $1 + .1$	**23.** $.1 + .1$	**24.** $10 + .1$	**25.** $1 + .01$
$1 - .1$	$.1 - .1$	$10 - .1$	$1 - .01$
$1 \times .1$	$.1 \times .1$	$10 \times .1$	$1 \times .01$
$1 \div .1$	$.1 \div .1$	$10 \div .1$	$1 \div .01$
26. $1.2 + 1.2$	**27.** $16 + .16$	**28.** $.15 + .15$	**29.** $1.2 + .04$
$1.2 - 1.2$	$16 - .16$	$.15 - .15$	$1.2 - .04$
1.2×1.2	$16 \times .16$	$.15 \times .15$	$1.2 \times .04$
$1.2 \div 1.2$	$16 \div .16$	$.15 \div .15$	$1.2 \div .04$
30. $2.4 + 2$	**31.** $6.3 + .3$	**32.** $4 + .05$	**33.** $.24 + .03$
$2.4 - 2$	$6.3 - .3$	$4 - .05$	$.24 - .03$
2.4×2	$6.3 \times .3$	$4 \times .05$	$.24 \times .03$
$2.4 \div 2$	$6.3 \div .3$	$4 \div .05$	$.24 \div .03$

34. George earned $2 one day and $.2 the next. How much did he earn in both days?

35. Richard had $2 and spent $.2. How much had he left?

36. Robert had $2 and bought neckties at $.2 each. How many did he buy?

37. Frank bought 2 baseballs at $.2 each. How much did he pay for them?

38. Mr. Atkinson had 5 acres of land. How much had he after buying .5 of an acre from his neighbor?

39. Mr. Blake had 5 acres and sold .5 of an acre. How much had he left?

40. Mr. Sayles sold .5 of a pasture containing 5 acres. How many acres did he sell?

41. A field containing 5 acres was cut up into lots of .5 of an acre each. How many lots?

LINEAR MEASURE *Oral*

A straight line is the shortest distance between two points.

A line has one dimension—length.

Linear or Long Measure is used in measuring lengths or distances.

1. Learn:

TABLE OF LINEAR MEASURE

12 inches (in.)	= 1 foot (ft.)
8 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet	= 1 rod (rd.)
320 rods	= 1 mile (mi.)

2. Fill the blanks:

1 mile = —— rods = —— yards = —— feet = —— inches

The sign ″ is frequently used for inches, and the sign ′ for feet. Thus, 2′ 6″ means 2 feet 6 inches.

3. How many inches in 1 ft. 3 in.? 2 ft. 8 in.? 3 ft. 4 in.? 4 ft. 2 in.? 5 ft.? 6 ft. 6 in.? $2\frac{3}{4}$ ft.? $1\frac{5}{6}$ ft.?

4. How many inches in $\frac{1}{2}$ yd.? $\frac{1}{4}$ yd.? $\frac{3}{4}$ yd.? $\frac{1}{8}$ yd.?

5. How many feet in $1\frac{1}{3}$ yd.? $2\frac{2}{3}$ yd.?

6. Express: 14 yards as feet; 14 feet as yards.

7. What part of a yard is 1 ft.? 2 ft.? $1\frac{1}{2}$ ft.? 1 ft. 3 in.? 1 ft. 8 in.? 2 ft. 6 in.?

8. Leon is 52 inches tall. How many feet?

9. How tall are you?

10. What is the height of the tallest boy in the room? Of the shortest boy? What is the difference in their heights?

11. How many inches from the floor is your desk? Your chair?

12. Make other measurements about the schoolroom.

Express as parts of a mile:

13.	160 rd.	80 rd.	40 rd.	20 rd.	10 rd.
14.	32 rd.	16 rd.	8 rd.	4 rd.	2 rd.

Change to rods:

15.	$\frac{1}{2}$ mi.	$\frac{1}{4}$ mi.	$\frac{3}{4}$ mi.	$\frac{5}{8}$ mi.	$\frac{3}{10}$ mi.
16.	$\frac{3}{16}$ mi.	$\frac{9}{20}$ mi.	$\frac{5}{32}$ mi.	$\frac{7}{40}$ mi.	$\frac{9}{80}$ mi.

PROBLEMS *Written*

1. How many telegraph poles, placed 10 rods apart, are required to string a mile of telegraph wire?

2. How much will the wire in problem 1 cost at $\frac{1}{2}$ of a cent a foot?

3. A gymnasium running track measures 16 laps to a mile. How many rods to a lap? What part of a mile does Victor run in making 5 circuits?

4. How many rods around a rectangular field 100 rods long and 60 rods wide? What name is given to this number of rods?

5. David runs a 100-yard dash in 12 seconds. How many feet a second?

6. A hand is 4 inches. A horse $14\frac{1}{2}$ hands high is how many feet high?

7. A passenger car is 70 feet long. How many yards? How many rods?

8. Blanche lives 3168 feet from the schoolhouse, Ruby 200 rods, and Alice $\frac{1}{2}$ of a mile. Who has the longest walk? Who has the shortest walk?

SQUARE OR SURFACE MEASURE

A surface has two dimensions — length and width. Square measure is used in measuring surfaces.

1. Learn:

TABLE OF SQUARE OR SURFACE MEASURE

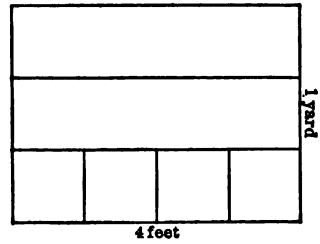
144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards or $272\frac{1}{4}$ square feet	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

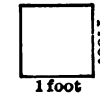
2. Fill the blanks:

1 A. = —— sq. rd. = —— sq. yd. = —— sq. ft.

The area of a surface is the number of square units it contains.

3. How many square feet in a rectangle 4 feet long and 1 yard wide?





4 feet by 1 yard = 4 feet by 3 feet.

What is the unit of measurement? How many of these units in 1 row? In all the rows?

Short method: $3 \times 4 = 12$.

Think first of the unit of measurement.

The area of a rectangle can always be found by multiplying together its length and its width, when both are expressed in the same *unit of measurement (inches, feet, yards, etc.).*

NOTE. While, for convenience, we say that we multiply the two dimensions together, it must be noted that this is not strictly true. We cannot say that 3 feet times 4 feet equals 12 square feet any more than we can say that 3 eggs times 4 eggs equals 12 square eggs, since the multiplier is always an abstract number and simply tells the number of times a quantity is taken. What we mean is that, since the unit of measurement is 1 square foot, we have in one row 4 of these units, or 4 square feet; in 3 rows we have 3 times 4 square feet, or 12 *square feet.*

PROBLEMS *Oral*

1. How many square inches in a rectangle 15″ by 6″?

2. How many square yards in a piece of canvas 12 yards long and $3\frac{1}{2}$ yards wide?

3. What is the area of a surface 3 yards long and 6 feet wide?

4. How long is a garden 4 yards wide, if it contains 60 square yards?

5. A square flower plot contains 36 square feet. What are its dimensions?

6. How many square feet in the top of a desk 4 feet long and $2\frac{1}{2}$ feet wide?

7. What is the area in square yards of a floor 12 feet long and 6 feet wide?

8. A square garden contains 81 square feet. How many square yards?

9. How many acres in a lot of land 32 rods long and 5 rods wide?

10. What part of an acre is a lot of land 8 rods by 2 rods? A lot 10 rods by 10 rods?

11. A piece of paper covers 96 square inches. It is 12 inches long. How wide is it?

12. A rectangle 12 feet long is twice as long as it is wide. Find its area.

13. How many rods of fence will inclose a lot 8 rods long and 6 rods wide?

14. What is the perimeter of a lot 8 rods square? The area?

NOTE. Much attention should be given to oral analysis of problems by pupils. This requires concentration of attention and a clear conception of the conditions stated. No set form of expression should be required, but the correct use of "if," "since," and "therefore" should be carefully taught. Pupils should be encouraged to be on the alert for short methods of solution, and to solve orally as many of the problems as possible.

Find the areas of rectangles of these dimensions:

	LENGTH	WIDTH		LENGTH	WIDTH
15.	16 in.	$12\frac{1}{2}$ in.	**16.**	1 ft. 6 in.	1 ft. 4 in.
17.	36 ft.	9 yd.	**18.**	$3\frac{1}{2}$ yd.	$2\frac{1}{2}$ ft.
19.	$3\frac{1}{2}$ ft.	30 in.	**20.**	$12\frac{1}{2}$ ft.	$12\frac{1}{2}$ ft.
21.	15 yd.	2 ft. 8 in.	**22.**	15 ft.	7 ft. 4 in.
23.	5 ft. 8 in.	$2\frac{1}{2}$ ft.	**24.**	12 rd.	66 ft.

25. Find the perimeters of the rectangles.

26. What is the area of the top of a table 4 feet 8 inches long and 2 feet 6 inches wide?

27. A rug is 144 inches long. Its width is two thirds of its length. How many square yards does it cover?

28. A lot of land containing 60 square rods cost $1272. How much would an acre cost at this rate?

29. How many acres in a lot of land 80 rods long and 24 rods wide?

Study these solutions:

(1) $80 \times 24 = 1920$, sq. rd. in lot.
$1920 \div 160 = 12$, A. in lot.

(2) $\dfrac{\overset{1}{\cancel{80}} \times \overset{3}{\cancel{24}}}{\underset{2}{\cancel{160}}} = 12$, A. in lot.

Note that in the mechanical work we treat the dimensions as abstract numbers.

Find the area in acres of lots of these dimensions:

	Length	Width		Length	Width
30.	25 rd.	18 rd.	**31.**	32 rd.	25 rd.
32.	48 rd.	45 rd.	**33.**	36 rd.	20 rd.
34.	$12\frac{1}{2}$ rd.	$12\frac{1}{2}$ rd.	**35.**	37.5 rd.	20 rd.
36.	$16\frac{2}{3}$ rd.	12 rd.	**37.**	8 rd.	66 ft.
38.	3.6 rd.	$2\frac{1}{2}$ rd.	**39.**	12.875 rd.	6.4 rd.

40. How many rods of fence would be required to inclose each lot?

41. How many acres in a lot 20 rods square?

42. What part of an acre is a lot containing 20 square rods?

43. The perimeter of a square lot is 100 rods. How many acres in the lot?

44. A square lot has an area of 25 square rods. Find the distance around it.

45. A field 80 rods by 45 rods was divided into 5 equal fields. How many acres in each lot?

46. How many acres in a lot one quarter of a mile long and one eighth of a mile wide? How many rods around the lot?

47. A park reservation 68 rods by 40 rods has 26 trees to the acre. How many trees in the reservation?

ANGLES

The difference in direction between two straight lines that meet is an *angle*.

The meeting point of the two straight lines is the *vertex* of the angle.

The lines are the sides of the angle.

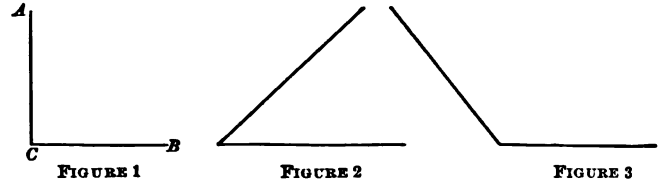


FIGURE 1 FIGURE 2 FIGURE 3

When the sides form a square corner, the angle is a *right angle.* (Fig. 1.)

When the sides form a right angle, they are *perpendicular* to each other. Thus in figure 1 the vertical line AC and the horizontal line BC are perpendicular to each other.

An angle less than a right angle is an *acute angle.* (Fig. 2.)

An angle greater than a right angle is an *obtuse angle.* (Fig. 3.)

QUADRILATERALS

A figure bounded by straight lines is a *polygon.*

A polygon of four sides is a *quadrilateral.*

Lines which have the same direction are *parallel lines.*

Quadrilaterals are classified according to their parallel sides:

(1) A quadrilateral whose opposite sides are parallel is a *parallelogram.*

(*a*) A parallelogram whose angles are right angles is a *rectangle.* (Fig. 1.)

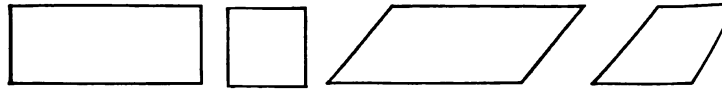


FIGURE 1 FIGURE 2 FIGURE 3 FIGURE 4

(*b*) A parallelogram whose sides are equal and whose angles are right angles is a *square.* (Fig. 2.)

(*c*) A parallelogram whose angles are oblique, *i.e.* not right angles, is an *oblique-angled parallelogram*, or a *rhomboid.* (Fig. 3.)

(*d*) An oblique-angled parallelogram whose sides are equal is a *rhombus.* (Fig. 4.)

FIGURE 5

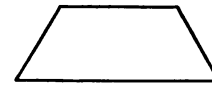


FIGURE 6

(2) A quadrilateral having two sides parallel is a *trapezoid.* (Figs. 5 and 6.)

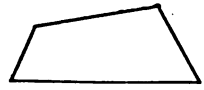


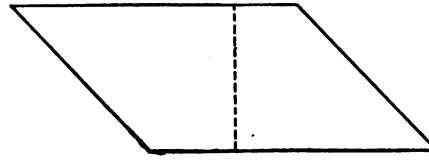
FIGURE 7

FIGURE 8

(3) A quadrilateral having no two sides parallel is a *trapezium.* (Figs. 7 and 8.)

FINDING AREAS OF PARALLELOGRAMS

1. Cut from paper a parallelogram like this — making base 4 inches and altitude 2 inches.



The altitude of a parallelogram is its height from its base.

2. Cut into two pieces and arrange the pieces to form a rectangle.

3. Compare the bases of the parallelogram and the rectangle.

4. Compare their altitudes.

5. What is the area of the rectangle? Of the parallelogram?

6. What must be known to find the area of a parallelogram?

The area of a parallelogram is equal to the product of its base and its altitude.

Find the areas of parallelograms of these dimensions:

	Length	Width		Length	Width
7.	18 ft.	$12\frac{1}{2}$ ft.	8.	15 in.	$4\frac{1}{2}$ in.
9.	$7\frac{1}{4}$ ft.	$5\frac{1}{2}$ ft.	10.	18 ft. 6 in.	12 ft.
11.	1′ 6″	1′ 6″	12.	3′ 3″	2′ 6″
13.	5 yd.	$8\frac{1}{2}$ ft.	14.	6 yd.	4 yd. 6 in.
15.	$5\frac{1}{4}$ ft.	$1\frac{2}{3}$ ft.	16.	.75 yd.	.5 yd.
17.	4.5 rd.	2.2 rd.	18.	48 ft.	15 yd.

19. Make up problems in finding areas of parallelograms.

FINDING AREAS OF TRAPEZOIDS

The altitude of a trapezoid is the perpendicular distance between its parallel sides.

1. What is the altitude of figure 1? Of figure 2?

2. Cut from paper a trapezoid like figure 1.

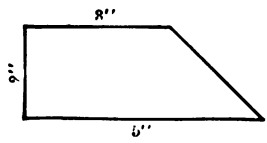


FIGURE 1

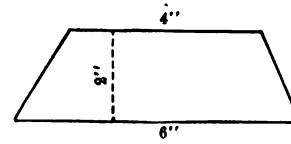


FIGURE 2

3. Fold so that the upper edge is on a line with the lower edge. Crease. Cut on the line of the crease. Arrange the two pieces to form a rectangle.

4. What two sides of the trapezoid form the base of the rectangle? What is the length of the base?

5. What is the altitude of the trapezoid? The altitude of the trapezoid is what part of the altitude of the rectangle?

6. What is the area of the rectangle? Of the trapezoid?

7. Cut from paper a trapezoid like figure 2. Fold, cut, and arrange the pieces to form a parallelogram.

8. What is the base of the parallelogram? Its altitude? Its area? What is the area of the trapezoid?

A trapezoid is equal to a parallelogram whose base is the sum of the parallel sides of the trapezoid, and whose altitude is one half the altitude of the trapezoid.

The area of a trapezoid is equal to one half the product of its altitude and the sum of its parallel sides.

9. What is the area of a trapezoid whose parallel sides are 24 feet and 16 feet, and whose altitude is 9 feet?

$24 + 16 = 40$, sum of parallel sides.

$\frac{40 \times 9}{2} = 180$, area in square feet.

Find the areas of trapezoids of these dimensions:

	Parallel Sides	Altitude
10.	10 ft. and 6 ft.	5 ft.
11.	18 in. and 12 in.	5 in.
12.	25 ft. and 15 ft.	10 ft.
13.	$2\frac{1}{2}$ ft. and $1\frac{1}{2}$ ft.	20 in.
14.	24 rd. and 16 rd.	14 rd.
15.	4 yd. and 3 yd.	2 yd.
16.	7.5 ft. and 4.5 ft.	5 ft.
17.	1.5 yd. and .8 yd.	.75 yd.

18. What is the area of a trapezoid if the base is 20 in., the side parallel to the base 12 in., and the altitude 8 in.?

19. The two parallel sides of a trapezoid are 16 ft. and 22 ft., and the altitude is 8 ft. Find the area.

20. How many square rods in a four-sided field whose two parallel sides are, respectively, 40 rods and 30 rods, and the distance between them is 15 rods?

21. Make a problem about a trapezoid. Draw the diagram, work out the problem, and ask your teacher to give the problem to the class to solve.

TRIANGLES

A polygon of three sides is a *triangle.*

Triangles may be classified in two ways:

I. With respect to their sides.

A triangle having three equal sides is an *equilateral triangle.* (Fig. 1.)

A triangle having two equal sides is an *isosceles triangle.* (Fig. 2.)

A triangle having no two sides equal is a *scalene triangle.* (Fig. 3.)

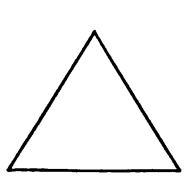


FIGURE 1

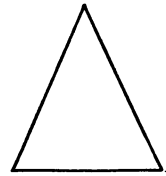


FIGURE 2

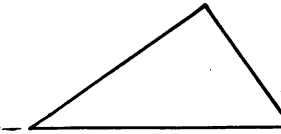


FIGURE 3

II. With respect to their angles.

A triangle having a right angle is a *right-angled triangle.* (Fig. 4.)

A triangle having three acute angles is an *acute-angled triangle.* (Fig. 5.)

A triangle having an obtuse angle is an *obtuse-angled triangle.* (Fig. 6.)

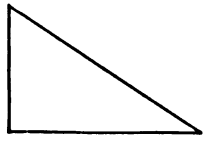


FIGURE 4

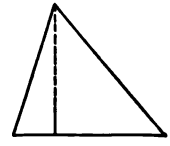


FIGURE 5

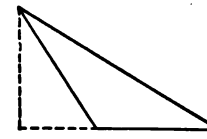


FIGURE 6

The altitude of a triangle is the perpendicular distance from the angle opposite the base to the base or to the base extended. What is the altitude of figure 4? Of figure 5? Of figure 6?

PROBLEMS *Written*

1. The perimeter of an equilateral triangle is 48 inches. What is the length of one side?

2. Each of the equal sides of an isosceles triangle is 15 inches and the base is 8 inches. What is the perimeter?

3. The perimeter of a right-angled triangle is 6 inches. The base is $1\frac{1}{2}$ inches and the altitude 2 inches. What is the length of the third side?

4. The perimeter of an isosceles triangle is 15 inches. The base is 3 inches. How long is each of the other sides?

5. The sides of a scalene triangle are $1\frac{1}{2}$ inches, $2\frac{3}{4}$ inches, and 3 inches. What is its perimeter?

6. A watch charm in the shape of an equilateral triangle requires a gold band $2\frac{1}{4}$ inches long to inclose it. What is the length of one side of the charm?

7. How many feet of wire fencing will inclose an equilateral flower bed, one of whose sides is 4.5 feet?

FINDING AREAS OF TRIANGLES

A straight line connecting the opposite corners of a quadrilateral is a diagonal. The line AB is a diagonal of figure 1.

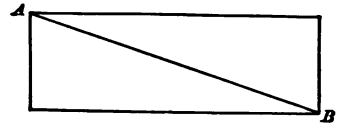


FIGURE 1

1. Cut from paper a rectangle whose base is 3 inches and whose altitude is 2 inches.

2. What is the area of this rectangle?

3. Cut along the diagonal. Compare the areas of the two triangles by placing one over the other.

4. The area of each triangle is what part of the area *of the rectangle*? What is the area of each triangle?

5. Cut from paper triangles like figure 2 and figure 3. Cut on dotted lines; arrange parts to form rectangles. Show that each triangle is equal to a rectangle whose base is equal to the base of the triangle and whose altitude is one half of the altitude of the triangle.

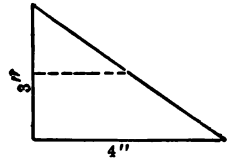


FIGURE 2

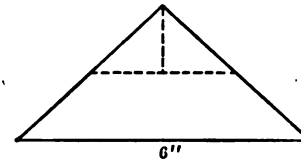


FIGURE 3

The area of a triangle is equal to one half the product of its base and its altitude.

Find areas of these triangles:

	Base	Altitude		Base	Altitude
6.	12 in.	5 in.	**7.**	18 in.	10 in.
8.	15 ft.	8 ft.	**9.**	20 ft.	15 ft.
10.	$2\frac{1}{2}$ ft.	15 in.	**11.**	1 ft. 8 in.	1 ft.
12.	5 yd.	3 yd.	**13.**	12 ft.	7.5 ft.
14.	3.5 yd.	3.5 ft.	**15.**	2.6 rd.	.75 rd.

16. The base of a right-angled triangle is 8 feet and the altitude is 5 feet. What is the area?

17. The base and altitude of a triangle are each 16 inches. What is the area?

18. What is the area of a triangular lot of land whose base is 20 yards and whose altitude is 25 feet?

19. How many acres in a triangular lot of land whose base and altitude are, respectively, 40 rods and 24 rods?

20. Make up problems in finding areas and perimeters of triangles.

DRAWING TO SCALE *Written*

1. Make a diagram of the top surface of the teacher's desk. Scale $\frac{1}{2}'' = 1'$.

2. Make a diagram of the side of the teacher's desk facing the school, on a scale of $\frac{1}{2}'' = 1'$.

3. Represent on paper the area of the schoolroom floor. Scale $1'' = 8'$.

Compute the number of square feet in the floor.

Compute the cost of the floor boards at 6 cents per square foot.

4. Make a diagram of a picture 24″ by 20″ surrounded by a 2″ frame. Scale $\frac{1}{4}'' = 2''$.

5. Make a diagram of the door leading from the schoolroom into the hall, on a scale of 1 inch = 1 foot.

6. Make a diagram of the lower sash of one of the schoolroom windows, letting 1 inch represent 6 inches. Compute the area of the lighting surface.

7. Draw a rectangle $2\frac{3}{4}$ inches by $1\frac{1}{2}$ inches. This figure represents the ground plan of a house drawn on a scale of $\frac{1}{4}$ inch to 4 feet. Find the dimensions of the house and the area covered by the house.

8. Four adjoining house lots are, respectively, 45 ft., 60 ft., 90 ft., and 75 ft. on the street side. Each lot is 120 ft. deep.

(*a*) Draw a diagram of the lots on a scale of 1 inch to 60 feet.

(*b*) Compute the values of the lots at $18\frac{3}{4}$ cents per *square foot.*

(*c*) What is the area of all the lots?

(*d*) What is the perimeter of each lot?

9. In many places building lots are sold by the street frontage. At $22.75 per front foot, what is the value of each lot in problem 8?

10. The diagram of a park covers a space $3\frac{1}{2}''$ by $5''$. $1''$ represents 24 rods. What are the dimensions of the park? Its area in acres?

11. On a scale of $\frac{1}{2}''$ to $1'$ make a diagram of a floor $16'$ by $12'$, and on it a rug 3 yd. by 2 yd. Compute the areas of the floor and of the rug in square feet.

12. Make a diagram of a mirror whose outside dimensions are 32 inches by 20 inches. Scale $\frac{1}{4}'' = 1$ foot.

13. Make a drawing of a 2-inch picture frame whose outside dimensions are 32 inches by 24 inches. Scale 1 inch to 8 inches.

14. Find from a geography map the distance from New York to Chicago. From Chicago to St. Louis. From St. Louis to Denver. From Denver to San Francisco.

15. Using some map in your geography, find the dimensions of the state of Colorado. Compute its area in square miles.

16. Find the dimensions and area of Wyoming. Of Kansas. Of Utah. Of Nevada. Of other states.

CUBIC OR VOLUME MEASURE *Oral*

A volume or solid has three dimensions — length, width, and thickness.

A solid bounded by six rectangles is a rectangular prism. (Fig. 1.)

If the six rectangles are squares, the solid is a cube. (Fig. 2.)

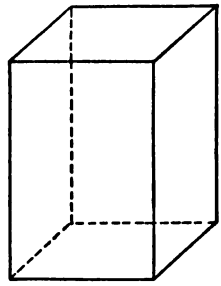


FIGURE 1

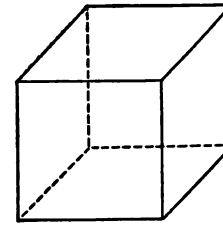


FIGURE 2

1. A 1-foot cube is a cube —— inches long, —— inches wide, and —— inches thick. It contains —— cubic inches.

2. A 1-yard cube is a cube —— feet long, —— feet wide, and —— feet thick. It contains —— cubic feet.

Cubic measure is used in measuring volumes.

3. Learn:

TABLE OF CUBIC MEASURE

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)

A load of earth, sand, etc., is a cubic yard.

4. How many inch cubes can be cut from a cubic foot of wood?

5. What is the volume of a 2-inch cube?

6. A 2-inch cube is how many times as large as 2 cubic inches?

7. How many 1-inch cubes are required to build a *4-inch cube?*

8. How many 2-inch cubes are equal in volume to a 4-inch cube?

9. How many blocks 1 inch × 1 inch × 1 inch can be packed in a box 4 inches × 2 inches × 3 inches?

10. A cubic foot is equal to how many 6-inch cubes?

11. A block of wood 8 inches long, 4 inches wide, and 2 inches thick will make how many 1-inch cubes?

12. A cake of ice 1 yard long, 1 yard wide, and 1 yard thick is cut into cakes 1 foot long, 1 foot wide, and 1 foot thick. How many?

13. A cubic foot is what part of a cubic yard?

14. Nine cubic feet are what part of a load of sand?

15. Thirty-six cubic yards are how many cubic feet?

16. Twenty-seven hundred cubic feet of earth are carted for filling. How many loads?

FINDING VOLUMES *Oral and Written*

The volume of a solid is the number of cubic units it contains.

1. What is the volume of a rectangular prism 4 inches long, 2 inches wide, and 6 inches high?

Figure 3 represents the prism. Figure 4 represents 1 cubic inch. The problem is to find how many cubic units like figure 4 there are in figure 3.

The unit of measurement is 1 cubic inch.

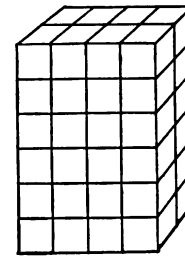


FIGURE 3

FIGURE 4

In the front row of the lowest layer there are 4 cubic inches.

In both rows of the layer there are 2 times 4 cubic inches, or 8 cubic inches.

In the six layers there are 6 times 8 cubic inches, or 48 cubic inches.

The steps taken are:

First. Determine the unit of measurement.

Second. Find the number of cubic units in 1 row.

Third. Multiply the number of cubic units in 1 row by the number of rows.

Fourth. Multiply the number of cubic units in 1 layer by the number of layers.

Think first of the unit of measurement.

The volume of a solid can always be found by multiplying together its length, its width, and its height, when all are expressed in the same *unit of measurement* (*inches, feet, yards, etc.*).

Find the volumes of these prisms:

	Length	Breadth	Height		Length	Breadth	Height
2.	10 ft.	3 ft.	6 ft.	**3.**	12 in.	6 in.	5 in.
4.	9 yd.	4 yd.	5 yd.	**5.**	4 ft.	$2\frac{1}{2}$ ft.	1 ft.
6.	3.5 ft.	2 ft.	1.5 ft.	**7.**	3 ft.	2.5 ft.	.5 ft.
8.	3 ft.	9 in.	6 in.	**9.**	4 yd.	9 ft.	2 ft.
10.	8 ft.	6 ft.	2 yd.	**11.**	9 ft.	2 yd.	5 ft.
12.	1 yd.	1 ft.	2 ft.	**13.**	27 in.	2 ft.	9 in.

14. Mr. Hubbard has in his barn an oat bin $6\frac{1}{2}$ feet long, 3 feet wide, and 4 feet deep. What is its capacity in cubic feet?

15. A wood box covers on the floor a space 2 ft. $\times 1\frac{1}{2}$ ft. It stands 2 ft. 4 in. high. How many cubic feet does the box occupy?

16. At 62.5 pounds to a cubic foot, what is the weight of the water contained in a tank 4 feet by 3 feet by 2 feet?

17. A cubic foot of ice weighs 57.5 pounds. What is the weight of a cake 2 feet long, 2 feet wide, and $1\frac{1}{2}$ feet thick?

18. How many loads of earth were removed in digging a cellar 24 feet long, 18 feet wide, and 6 feet deep?

Study these solutions:

(1) $24 \times 18 \times 6 = 2592$ — The product of the dimensions expressed in feet represents the contents in cubic feet.

$2592 \div 27 = 96$ — The number of cubic feet of earth removed divided by the number of cubic feet in one load gives the number of loads.

(2) $\dfrac{\overset{2}{24} \times \overset{2}{18} \times 6}{\underset{3}{27}} = 96$

$24 \times 18 \times 6$ represents what?

$\dfrac{24 \times 18 \times 6}{27}$ represents what?

In problem work always select the shortest method.

Find the number of loads of earth removed in digging a cellar:

19. 18 feet by 21 feet by 6 feet.

20. 24 feet by 30 feet by 6 feet.

21. 36 feet by 30 feet by 7 feet.

22. 28 feet by 36 feet by 9 feet.

23. Baseballs are put up in pasteboard cases 3 inches long, 3 inches wide, and 3 inches high. How many can be packed in a box 18 inches long, 15 inches wide, and 12 inches deep?

24. A pile of wood 8 feet long, 4 feet wide, and 4 feet high is 1 cord. How many cubic feet in a cord?

25. One eighth of a cord is a cord foot. How many cubic feet in a cord foot?

26. Learn:

TABLE OF WOOD MEASURE

16 cubic feet = 1 cord foot (cd. ft.)
8 cord feet (128 cubic feet) = 1 cord (cd.)

27. How many cords in a pile of wood 16 feet by 4 feet by 4 feet?

28. How many cord feet in a pile of wood 4 feet by 4 feet by 4 feet?

29. At $1.25 per cord foot, what is a cord of maple wood worth?

30. Mr. Baldwin bought 3 cord feet of pine wood for $1.50. What was the rate per cord?

31. How many cords of spruce in a pile containing 704 cubic feet?

32. How many cords of hard wood in a pile containing 704 cord feet?

Find the number of cords of wood in piles of these dimensions:

	Length	Width	Height		Length	Width	Height
33.	8 ft.	4 ft.	8 ft.	**34.**	15 ft.	4 ft.	6 ft.
35.	20 ft.	4 ft.	12 ft.	**36.**	12 ft.	4 ft.	10 ft.
37.	25 ft.	4 ft.	8 ft.	**38.**	18 ft.	4 ft.	6 ft.

39. At $8 a cord, what is the value of a pile of wood 12 feet long, 4 feet wide, and 9 feet high?

40. How many cords of bark in a pile 16 feet long, 8 feet wide, and 10 feet high?

41. A freight car 30 feet long has on it a pile of pine wood 8 feet wide and 8 feet high. How many cords?

SURFACES OF RECTANGULAR PRISMS

1. Draw on cardboard or stiff paper a diagram like figure 2. Cut it out. Crease on dotted lines and fold to make a rectangular prism like figure 1. Sew, pin, or paste the edges together.

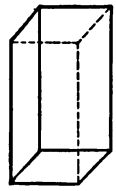


FIGURE 1

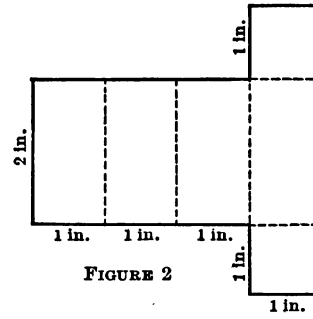


FIGURE 2

How many sides or faces has this prism? What is the area of the top and bottom faces? Of the front and back faces? Of the two side faces? Of the six faces?

2. Draw a diagram showing the faces of a 2-inch cube.

3. What is the surface area of a 5-inch cube? Of a 6-inch cube?

4. Represent by diagram the surface of a block of wood 2 inches by 2 inches by 3 inches. What is the surface area?

5. Make a diagram showing the surface of a brick 8 inches long, 4 inches wide, and 2 inches thick. Scale $1'' = 2''$. What is the surface area?

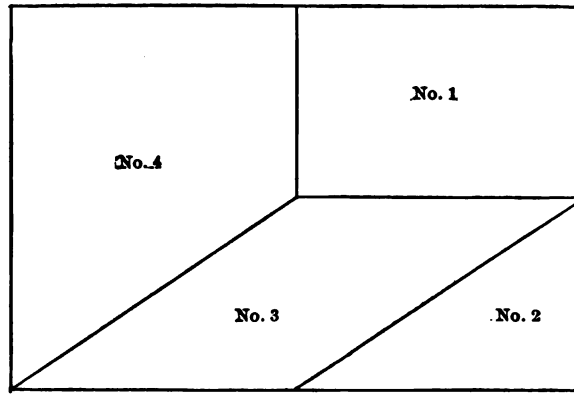
6. What is the surface area of a box 12 inches by 9 inches by 8 inches? Scale $1'' = 3''$.

7. Find the surface areas of the prisms in examples 2 to 13 on page 104.

8. Make a diagram of the outside surface of a rectangular collar box without cover. Of an envelope box.

REVIEW EXERCISE *Written*

1. This diagram represents a field divided into four lots. Copy it, and answer these questions about it.



Scale $\frac{1}{2}'' = 20$ rods

(1) What kind of figure is lot No. 1? What are its dimensions? What is its area in acres?

(2) What kind of figure is lot No. 2? What are its dimensions? Its area in acres?

(3) What name is given to a figure like lot No. 3? What is the length of its base? Its altitude? How many acres in the lot?

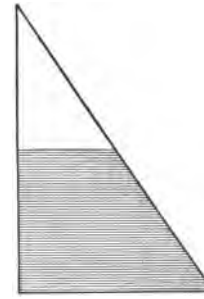
(4) What kind of figure is lot No. 4? What dimensions must be known to find its area? Find its area in acres.

(5) Find the sum of the areas of the lots. Find a way of testing your answer.

(6) What is the perimeter of the field in rods? In feet? In miles?

2. Draw a right-angled triangle, an acute-angled triangle, and an obtuse-angled triangle whose bases are 4 inches and whose altitudes are 3 inches. Compare their areas.

3. What is the area of the shaded part of this figure? The unshaded part? What is the sum of the two parts? Test the last answer by finding the area of the figure in another way.



Scale $\frac{1}{4}'' = 6'$

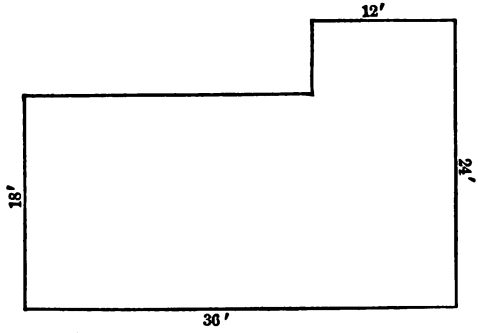
4. Draw a diagram showing the surface area of a block of wood 2 inches square and 3 inches high. Compute the area.

5. The following diagram represents a cellar floor.

(1) Draw a plan of it, letting $\frac{1}{2}$ inch represent 3 feet.

(2) What is the perimeter of the plan? How many feet does it represent? How many yards?

(3) What is the area in square feet of the cellar floor? In square yards?



(4) The cellar was excavated to a depth of 6 feet. How many cubic feet of earth were removed? How much did the labor cost at $1 a load?

(5) A cement floor 3 inches thick was laid in the cellar. How many cubic feet of material were used? How many cubic yards?

LIQUID MEASURE *Oral and Written*

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)

1. A pint is what part of a quart? Of a gallon?
2. A quart is what part of a gallon?
3. What part of a gallon is 2 quarts? 3 quarts?
4. At 4 cents a pint a quart of milk will cost —— cents, and a gallon will cost —— cents.
5. At 90 cents a quart, what will a gallon of olive oil *cost? A pint?*

6. Express 15 quarts as gallons; as pints.

7. Express 17 pints as quarts; as gallons.

8. How many pints in 2 gal. 3 qt.? In $5\frac{3}{4}$ gal.?

9. How many pints in 2 gal. 1 qt.? In 1 gal. 1 qt. 1 pt.?

10. How many pints of cream in a gallon-jar $\frac{3}{4}$ full? At $3\frac{1}{2}$ cents a quart, what is received for a can of milk containing 2 gallons 1 quart?

11. How many quart jars can be filled from a kettle containing $2\frac{3}{4}$ gallons of preserves?

12. How many gallons of sirup in a dozen bottles each holding $\frac{7}{8}$ of a quart?

13. A barrel of vinegar ($31\frac{1}{2}$ gallons) is to be put up in half-gallon bottles. How many bottles will be required?

14. A 5-gallon carboy of spring water costs 25 cents. What is the rate per quart?

15. At 64 cents a gallon, what is the cost of $1\frac{1}{2}$ pints of machine oil?

16. At 5 cents a quart, how much must be paid for 1 gallon 2 quarts of vinegar?

17. How many gallons in 2 dozen bottles, each holding 1 quart 1 pint?

18. How many half-pint bottles can be filled from a jug containing 2 gallons 2 quarts 2 pints?

19. Mr. Chapin uses a gallon of gasolene in driving his automobile 18 miles. How far can he go if he has 4 gallons 3 quarts of gasolene?

20. At 15 cents a gallon, how much does it cost Mr. Chapin to run his automobile 1 mile?

DRY MEASURE *Oral and Written*

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

1. Express as bushels: 20 pecks; 2 bu. 4 qt.; 1 bu. 7 qt.; 72 qt.

2. Express as pecks: $\frac{5}{8}$ bu.; 1 bu. 3 pk.; 15 qt.; 4 bu.

3. Express as quarts: $1\frac{3}{4}$ pecks; 2 bu. 3 pk.; $2\frac{3}{4}$ pk.; 4 bu.

4. At 2 cents a quart a peck of corn will cost —— cents, and a bushel will cost —— cents.

5. A quart is what part of a peck? Of a bushel?

6. What part of a peck is 1 quart? 3 quarts? 5 quarts? 7 quarts? 2 quarts? 4 quarts? 6 quarts?

7. What part of a bushel is 1 peck? 3 pecks? 2 pecks?

8. A bushel of oats weighs 32 pounds. What is the weight of a peck? A quart? $1\frac{3}{4}$ bushels? $2\frac{1}{2}$ pecks?

9. A bushel of corn weighs 56 pounds. What is the weight of a peck? A quart? $1\frac{1}{2}$ bushels?

10. A bushel of wheat weighs 60 pounds. What is the weight of a peck? A quart? $\frac{3}{4}$ of a bushel? $2\frac{1}{2}$ bushels?

11. At 12 cents a quart, what will $2\frac{1}{2}$ pecks of cranberries cost?

12. If a berry basket holds $\frac{3}{5}$ of a quart, how many baskets can be filled from a bushel of blueberries?

13. How many boxes, each containing $2\frac{1}{2}$ pecks, can be *filled from* a bin holding 25 bushels of apples?

AVOIRDUPOIS WEIGHT *Oral*

1. Learn:

TABLE OF AVOIRDUPOIS WEIGHT

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
2000 pounds	= 1 ton (T.)
2240 pounds	= 1 long ton

The long ton is used in measuring coal and ore at the mines, etc.

2. How many pounds in 25 ounces? In 60 ounces?

3. How many ounces in $\frac{1}{2}$ of a pound? $\frac{1}{4}$? $\frac{3}{4}$? $\frac{1}{8}$? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{7}{8}$? $\frac{1}{16}$? $\frac{5}{16}$? $\frac{7}{16}$?

4. How many 100-pound bags of coal in 1 ton?

5. How many tons in 2500 pounds? 3000 pounds? 5000 pounds? 7500 pounds?

Express as fractions of a ton:

6. 1000 pounds 500 pounds 1500 pounds 400 pounds

7. 800 pounds 1200 pounds 1600 pounds 200 pounds

PROBLEMS *Written*

1. A piece of butter weighs 8 pounds 10 ounces. How much is it worth at 36 cents a pound?

2. At 20 cents a pound, what is the cost of a piece of cheese weighing 20 ounces?

3. What is the rate per ton when a 100-pound bag of fertilizer costs $1.75?

4. Make out Mr. Alfred S. Hosmer's bill for .75 of a ton of fertilizer at $36 a ton.

5. At the rate of $18 a ton, how much will 2 tons 12 hundred pounds of hay cost?

6. How many ordinary tons in a shipment of 30,000 pounds of coal? How many long tons?

7. A farmer received $40 for $1\frac{1}{2}$ tons of potatoes, selling them by the bushel of 60 pounds. How much did he receive per bushel?

8. A cubic foot of water weighs 1000 ounces. How many pounds?

9. How many cubic feet of water weigh 1 ton?

4200
3750
4750
4840
4290
4370
4760
5860
5480
4530
4920
4250

10. These numbers represent the weight in pounds of twelve loads of coal delivered at a schoolhouse.

(1) What was the weight of each load in tons?

(2) What was the total weight in pounds? In tons?

(3) Find the cost at $5.84 a ton.

(4) How many long tons in all the loads?

11. Mr. Bartlett bought a ton of fertilizer for $37.50, and retailed it by the hundredweight at the rate of $2\frac{1}{2}$ cents a pound. What was his profit?

PERCENTAGE *Oral*

1. How many cents in a dollar?

2. How many hundredths of a dollar is 1 cent? 10 cents? 25 cents? 50 cents? 62 cents? 87 cents? 100 cents?

3. Into how many hundredths can a mile be separated? An acre of land? A bushel of grain? A barrel of flour? Any thing?

When we think of a thing as divided into 100 equal parts, we speak of $\frac{1}{100}$, or .01, of it as 1 *per cent* of it; .05 of it as 5 per cent of it; .50 of it as 50 per cent of it; and so on.

The expression *per cent* is an abbreviation of the Latin phrase *per centum*, meaning *by the hundred.*

Reckoning by the hundred is such an easy way of computation that it is largely used in business.

The sign % means the same as the words per cent. Thus, 25 per cent and 25 % mean the same thing.

There is no principle in percentage not already used in integers and fractions. In percentage we are simply thinking of things as made up of 100 equal parts, or 100 hundredths.

4. What per cent of a century is $\frac{1}{100}$ of it? $\frac{2}{100}$ of it? $\frac{67}{100}$ of it?

5. What per cent of a mile is .01 of it? .17 of it? .56 of it?

6. What per cent of any thing is $\frac{16}{100}$ of it? .25 of it? $\frac{100}{100}$ of it?

7. What per cent of 1 yard is 1 yard? What per cent of a yard is $\frac{1}{2}$ of a yard? $\frac{1}{4}$ of a yard? $\frac{3}{4}$ of a yard?

8. What per cent of 1 is 1? What per cent of 1 is 2? What per cent of 1 is 4?

9. What per cent of a thing is the whole of it? $\frac{1}{2}$ of it? $\frac{1}{4}$ of it? $\frac{1}{5}$ of it? $\frac{1}{10}$ of it?

10. What per cent of 1 is $\frac{1}{2}$? What per cent of 1 is $\frac{1}{4}$? $\frac{3}{4}$?

11. Grace spelled 90 per cent of the words in the spelling lesson. What per cent of the words did she misspell?

12. The grocer sold a barrel of flour at a loss of 10 per cent. What per cent of the cost did he receive?

13. Foxes killed 25 per cent of Mr. Marston's chickens. What per cent was left?

14. If he had 60 at first, how many were killed? How many were left?

15. Express these per cents decimally:

1 % 7 % 9 % 21 % 37 % 63 % 71 % 93 %

16. Express these decimals with the sign %:

.01 .03 .11 .17 .41 .67 83 .97

FRACTIONS AS EQUIVALENT PER CENTS

1. How many hundredths of a dollar is the whole of it? What per cent of it?

2. $\frac{1}{2}$ of a dollar is how many hundredths of it? What per cent of it?

Express as hundredths and as per cents:

Thus, $\frac{1}{2} = \frac{50}{100} = .50 = 50\ \%$; $\frac{1}{3} = \frac{33\frac{1}{3}}{100} = .33\frac{1}{3} = 33\frac{1}{3}\ \%$.

3. $\frac{1}{4}$	**4.** $\frac{3}{4}$	**5.** $\frac{4}{4}$	**6.** $\frac{1}{5}$	**7.** $\frac{2}{5}$
8. $\frac{3}{5}$	**9.** $\frac{4}{5}$	**10.** $\frac{5}{5}$	**11.** $\frac{1}{10}$	**12.** $\frac{3}{10}$
13. $\frac{7}{10}$	**14.** $\frac{9}{10}$	**15.** $\frac{10}{10}$	**16.** $\frac{1}{20}$	**17.** $\frac{3}{20}$
18. $\frac{7}{20}$	**19.** $\frac{11}{20}$	**20.** $\frac{20}{20}$	**21.** $\frac{1}{25}$	**22.** $\frac{6}{25}$
23. $\frac{18}{25}$	**24.** $\frac{21}{25}$	**25.** $\frac{25}{25}$	**26.** $\frac{3}{50}$	**27.** $\frac{50}{50}$

28. $\frac{1}{3}$	**29.** $\frac{2}{3}$	**30.** $\frac{3}{3}$	**31.** $\frac{1}{6}$	**32.** $\frac{5}{6}$
33. $\frac{6}{6}$	**34.** $\frac{1}{8}$	**35.** $\frac{3}{8}$	**36.** $\frac{5}{8}$	**37.** $\frac{7}{8}$
38. $\frac{8}{8}$	**39.** $\frac{1}{12}$	**40.** $\frac{12}{12}$	**41.** $\frac{1}{16}$	**42.** $\frac{16}{16}$

PER CENTS AS EQUIVALENT FRACTIONS

Express these per cents as common fractions in their lowest terms; thus, $50\,\% = \frac{50}{100} = \frac{1}{2}$; $33\frac{1}{3}\,\% = \frac{33\frac{1}{3}}{100} = \frac{1}{3}$:

1. 50 %	**2.** 25 %	**3.** 75 %	**4.** 20 %	**5.** 40 %
6. 60 %	**7.** 80 %	**8.** 10 %	**9.** 30 %	**10.** 70 %
11. 90 %	**12.** 5 %	**13.** 4 %	**14.** 2 %	**15.** 1 %
16. $33\frac{1}{3}$ %	**17.** $66\frac{2}{3}$ %	**18.** $16\frac{2}{3}$ %	**19.** $83\frac{1}{3}$ %	**20.** $12\frac{1}{2}$ %
21. $37\frac{1}{2}$ %	**22.** $62\frac{1}{2}$ %	**23.** $87\frac{1}{2}$ %	**24.** $8\frac{1}{3}$ %	**25.** $6\frac{1}{4}$ %

PER CENTS EXPRESSED IN DIFFERENT WAYS

Five per cent may be expressed in five different ways: (1) 5 per cent; (2) 5 %; (3) .05; (4) $\frac{5}{100}$; (5) $\frac{1}{20}$. Write these per cents in different ways:

1. 10 %	**2.** 20 %	**3.** 25 %	**4.** 30 %	**5.** 40 %
6. 50 %	**7.** 60 %	**8.** 70 %	**9.** 75 %	**10.** 80 %
11. 90 %	**12.** 6 %	**13.** 8 %	**14.** 15 %	**15.** 32 %
16. 55 %	**17.** 72 %	**18.** 84 %	**19.** 85 %	**20.** 96 %

Fill the blanks: *Oral*

21. $10\,\% = \frac{1}{10} = \frac{2}{20} = \frac{3}{30} = \frac{4}{40} = \frac{5}{50} = \frac{6}{60} = \frac{7}{70} = \frac{8}{80}$.

22. $50\,\% = \frac{1}{2} = \frac{4}{\ } = \frac{6}{\ } = \frac{8}{\ } = \frac{10}{\ } = \frac{12}{\ }$ and so on to $\frac{50}{100}$.

23. $25\,\% = \frac{1}{4} = \frac{8}{\ } = \frac{12}{\ } = \frac{16}{\ } = \frac{20}{\ } = \frac{24}{\ }$ and so on to $\frac{25}{100}$.

24. $20\,\% = \frac{1}{5} = \frac{10}{\ } = \frac{15}{\ } = \frac{20}{\ } = \frac{25}{\ }$ and so on to $\frac{20}{100}$.

25. In like manner write the fractional equivalents of other per cents.

Some per cents have such simple fractional equivalents that they should be memorized and used whenever possible in solving problems. The following table gives the more common ones.

TABLE OF EQUIVALENTS

$50\% = \frac{1}{2}$	$20\% = \frac{1}{5}$	$10\% = \frac{1}{10}$	$33\frac{1}{3}\% = \frac{1}{3}$	$12\frac{1}{2}\% = \frac{1}{8}$
$25\% = \frac{1}{4}$	$40\% = \frac{2}{5}$	$30\% = \frac{3}{10}$	$66\frac{2}{3}\% = \frac{2}{3}$	$37\frac{1}{2}\% = \frac{3}{8}$
$75\% = \frac{3}{4}$	$60\% = \frac{3}{5}$	$70\% = \frac{7}{10}$	$16\frac{2}{3}\% = \frac{1}{6}$	$62\frac{1}{2}\% = \frac{5}{8}$
	$80\% = \frac{4}{5}$	$90\% = \frac{9}{10}$	$83\frac{1}{3}\% = \frac{5}{6}$	$87\frac{1}{2}\% = \frac{7}{8}$

NOTE. Drill should be given on the above table until equivalents are mastered.

FINDING THE PERCENTAGE OF A NUMBER

Oral and Written

The per cent of a number or a quantity is the *percentage*. Thus, 10 % of 80 is 8; 8 is the percentage. 25 % of $20 is $5; $5 is the percentage.

1.

10	20	30	40	50
120	Find **10 %** of			60
110	100	90	80	70

In the center space use 20 %; 30 %; 40 %; 50 %; 60 %; 70 %; 80 %; 90 %.

2.

16	40	64	80	24
8	Find **25 %** of			48
88	56	32	96	72

In the center space use $12\frac{1}{2}$ %; 25 %; $37\frac{1}{2}$ %; $62\frac{1}{2}$ %; 75 %; $87\frac{1}{2}$ %.

3.

12	30	6	48	66
72	Find **50 %** of			18
60	42	24	54	36

In the center space use $16\frac{2}{3}\%$; $33\frac{1}{3}\%$; $66\frac{2}{3}\%$; $83\frac{1}{3}\%$.

4. Find:

10%	$16\frac{2}{3}\%$	20%	25%	30%	$33\frac{1}{3}\%$	40%
100%	**of 60 minutes**					50%
90%	$83\frac{1}{3}\%$	80%	75%	70%	$66\frac{2}{3}\%$	60%

Write in the center space \$1.20; 240 acres of land; \$300; 420 barrels of oil; and so on.

5. Find:

100%	10%	$12\frac{1}{2}\%$	20%	25%	30%	$37\frac{1}{2}\%$
90%	**of \$0.40**					40%
$87\frac{1}{2}\%$	80%	75%	70%	$62\frac{1}{2}\%$	60%	50%

Write in the center space 80 bushels; 160 square rods; \$200; 320 rods; 360 days; and so on.

Make up problems based on the combinations given in the five exercises above.

PROBLEMS *Oral*

1. Carl has completed 60 % of his work. How much remains to be done?

2. Mary's record in a spelling test was 90 %. There were 50 words. How many did she spell correctly?

3. Charles paid 50 cents for a knife. He sold it for 20 % less than he paid. How much did he lose?

4. Edward paid 60 cents for his knife. He sold it for 25 % more than he paid. How much did he gain?

5. A baseball team's standing is given as 75 %. What does this mean? How many of the 48 games played has it won?

6. A loaf of bread costs the baker 4 cents. He sells it at a gain of 25 %. How much does he receive for it?

7. John has 60 marbles and Will has 25 % more. How many has Will?

8. One automobile travels at the rate of 30 miles an hour and another 80 % as fast. What is the rate of the second?

9. Max sold 72 newspapers on Thursday and $12\frac{1}{2}$ % more on Friday. How many did he sell on Friday?

10. Ellen bought 12 yards of cloth. She used $66\frac{2}{3}$ % of it in making a dress. How many yards did she use?

11. Mr. Morris gathered 12 bushels of tomatoes from his vines. He sold 50 % of them to the grocer, 25 % of them to neighbors, and kept the rest for his own use. How many bushels did he keep?

PROBLEMS *Written*

1. Mr. Howes had a farm of 128 acres. He sold 75 per cent of it. How many acres did he sell?

Study these three solutions:

(1) 128
.75
96.00

To find 75 % of a number is to find .75 of it; that is, to multiply the number by .75.

(2) 1.28
75
96.00

1 % of 128 is 1.28. 75 % of 128 is 75 times 1.28, or 96.

(3) 4)128
32
3
96

75 % = .75 = $\frac{75}{100} = \frac{3}{4}$. $\frac{3}{4}$ of 128 = 96.

2. Seventy-five per cent of a park of 48 acres was made into a playground. What was the area of the playground?

3. A suit of clothes cost $18. A pair of shoes cost 15 per cent as much. How much did the shoes cost?

4. Twenty per cent of the 180 geographies in a school are worn out. How many new ones must be bought to take their place?

5. Mrs. Adams bought a 56-pound tub of butter at 25 cents a pound. 8 per cent of the butter spoiled. What was the money loss?

6. Mr. Harrow raised 144 bushels of turnips. He sold 25 per cent of them. How many bushels did he sell?

7. Mr. Francis had 144 acres in his farm. He sold 27 per cent of them. How many did he sell?

8. Of the 450 pupils in a school 42 per cent are boys. What per cent are girls? How many girls in the school?

9. To reduce his stock, a merchant makes these reductions:

Article	Regular Price	Reduction	Article	Regular Price	Reduction
Sleds	$1.50	10 %	Skates	$1.40	15 %
Dolls	.88	25 %	Doll Carriages	1.38	$16\frac{2}{3}$ %
Games	.40	$12\frac{1}{2}$ %	Magic Lanterns	2.00	$37\frac{1}{2}$ %
Carts	2.70	30 %	Toy Engines	1.80	50 %
Knives	.85	20 %	Picture Puzzles	.75	$33\frac{1}{3}$ %

(1) How much is taken off the regular price on each article?

(2) What is the sale price of each article?

FINDING WHAT PER CENT ONE NUMBER IS OF ANOTHER

Oral and Written

1. What part of 100 is 1? 3? 7? 19? 33?

2. How many hundredths of 100 is 9? 13? 27?

3. What per cent of 100 is 5? 8? 25? 45? 50?

4. What part of 20 is 5? How many hundredths of 20 is 5? What per cent of 20 is 5?

5. What part of 20 is 15? How many hundredths? What per cent?

6. What per cent of \$10 is \$1? \$2? \$3? \$4? \$5? \$6? \$7? \$8? \$9? \$10?

7. What part of a dozen oranges is 6 oranges? Express the answer (1) as a common fraction, (2) as a decimal, and (3) as a per cent.

8. What per cent of a gallon is 1 qt.? 2 qt.? 3 qt.? 4 qt.?

9. What per cent of a gallon is 1 pt.? 3 pt.? 2 qt. 1 pt.? $3\frac{1}{2}$ qt.?

10. What per cent of a foot is:

2 inches?	3 inches?	4 inches?	6 inches?
8 inches?	9 inches?	10 inches?	12 inches?

11. What per cent of a day is represented by:

3 hours?	4 hours?	6 hours?	8 hours?
9 hours?	12 hours?	15 hours?	16 hours?
18 hours?	20 hours?	21 hours?	24 hours?

Tell what per cent of:

12. 14 is 7	**13.** 12 is 4	**14.** 30 is 6	**15.** 40 is 5
16. 36 is 9	**17.** 60 is 6	**18.** 48 is 8	**19.** 16 is 6
20. 24 is 24	**21.** 24 is 16	**22.** 27 is 18	**23.** 60 is 15
24. 40 is 16	**25.** 20 is 12	**26.** 27 is 27	**27.** 27 is 18
28. 60 is 48	**29.** 32 is 20	**30.** 56 is 49	**31.** 30 is 9
32. 40 is 36	**33.** 60 is 42	**34.** 36 is 30	**35.** 40 is 40

36. Express as per cents of an hour:

(1) 60 min. (2) 15 min.; 45 min.
(3) 30 min. (4) 20 min.; 40 min.
(5) 3 min. (6) 10 min.; 50 min.
(7) 12 min.; 18 min.; 36 min.; 48 min.
(8) 6 min.; 18 min.; 42 min.; 54 min.
(9) 9 min.; 21 min.; 27 min.; 51 min.

PROBLEMS *Oral*

1. Two boys mowed a lawn for 40 cents. They shared the gain equally. What per cent of the pay did each get?

2. Two men plowed a field. One plowed $\frac{1}{4}$ of it and the other $\frac{3}{4}$. What per cent of the pay should each receive?

3. James is 3 years old. Robert is 15 years old. What per cent of Robert's age is James's age?

4. Clarence collected $20 for a newspaper company. They paid him $2. This was what per cent of the money collected?

5. Blanche solved 8 of the 10 examples given to the class. What per cent?

6. Four boys hoed a patch of potatoes. Albert worked 4 hours, Sam 3 hours, Ernest 2 hours, and Max 1 hour. What per cent of the work did each do?

7. The 40 pupils in an evening school are engaged as follows: 4 are studying drawing, 6 stenography, 8 typewriting, 10 manual training, and 12 grammar. What per cent of the pupils are engaged in each study?

8. Frances earned $25. She bought a dress for $12, a hat for $4, a pair of shoes for $3, and put $6 in the bank. What per cent of her $25 did she pay for each article? What per cent of her earnings did she save?

9. The Washington, Hamilton, Jefferson, and Lincoln schools played a series of ball games. Of the games played the Washingtons won 4, the Hamiltons 6, the Jeffersons 7, and the Lincolns 3. Express the standing of each club as a per cent.

PROBLEMS *Written*

1. Mr. Billings sold 21 of the 56 bushels of corn that he raised. What part of his crop did he sell? What per cent?

Study these solutions:

(1) $\frac{21}{56} = \frac{3}{8} = .37\frac{1}{2} = 37\frac{1}{2}\%$

(2) $56\overline{)21.000}$ $\quad .375 = .37\frac{1}{2} = 37\frac{1}{2}\%$

2. A farmer had 75 sheep and sold 36 of them. What per cent of his flock did he sell?

3. Eighteen of the 120 trees in a wood lot were cut *down*. This was what per cent of the whole number?

4. Of the 150 people on a picnic, 90 are children. The children are what per cent of the party?

5. Of the 400 pupils in a school, 250 are girls. The girls are what per cent of the pupils in the school?

6. \$16 is what per cent of \$80? \$160 is what per cent of \$800? \$1600 is what per cent of \$8000?

7. Mr. Maxwell sold 48 acres of his 120-acre farm. What per cent of his farm did he sell?

8. If a man owes \$360, and has \$270, what per cent of his debts can he pay?

9. A farmer raised 240 bushels of potatoes in one field and 360 bushels in another. What per cent of the whole crop did he raise in the first field? In the second?

10. In one season Mr. Martin raised 600 quarts of strawberries. He sold 210 quarts in town and shipped the rest to the city. What per cent did he sell in town? What per cent did he ship to the city?

11. A clerk with a salary of \$1200 saves \$300. What per cent of his salary does he save?

12. If a man earns \$1500 and spends \$1200, what per cent of his money does he save?

13. Mr. Clark built a house for \$2400. He rents it for \$240 a year. What per cent of the cost of the house is the yearly rent?

14. Mr. Blake built a house for \$2400 which he rents for \$25 a month. What per cent of the cost of the house is the yearly rent?

15. What per cent of a mile is a street 128 rods long?

16. MARK-DOWN SALE AT A LINEN STORE

Article	Regular Price	Sale Price
Tablecloths, 2 × 2 yd.	$2.50	$2.00
Tablecloths, 2 × 3 yd.	3.75	2.50
Napkins (dozen), 22 × 22 in.	3.50	2.80
Napkins (dozen), 24 × 24 in.	3.00	2.50
Towels (turkish)	.50	.37½
Damask tray cloths	.75	.50
Pillow slips	.24	.15
Blankets (pair)	5.00	3.75
Sheets	.90	.75
Handkerchiefs	.25	.12½

(1) The sale price of each article was how much less than the regular price?

(2) The amount deducted in each case was what per cent of the regular price?

ORAL PROBLEMS

1. A merchant buys a mowing machine for $40.

(*a*) If he sells it for $48, how much will he gain? What part of the cost? What per cent of the cost?

(*b*) If he sells it for $36, how much will he lose? What part of the cost? What per cent of the cost?

(*c*) How much will he gain if he sells it at a gain of 25%? What will be the selling price?

2. Mr. Sanderson owes a bill of $480. If he pays 12½% of it a week, how long will it take to pay the bill? How much will he pay each week?

3. What per cent is gained when a horse that cost *$200 is sold* at a gain of $40?

4. A huckster buys bananas at 15 cents a dozen and sells them at 20 cents a dozen. What per cent does he gain?

5. Paul had a collection of 60 coins. How many had he after buying 20 % more?

6. How many more days in September after $83\frac{1}{3}$ % of the month has passed?

7. A merchant buys hammers at 30 cents apiece and sells them at 40 cents apiece. What per cent does he gain?

8. If 24 quarts of oats and 8 quarts of corn are mixed together, what per cent of the mixture is corn? What per cent oats?

9. Paul earns 80 cents a day. How much will he receive if his wages are raised 20 %?

10. What per cent of perfect rank does Alice get when she spells 24 of the 25 words in the lesson?

11. Mr. Carlton raised 72 bushels of corn. He sold 25 per cent of it, lost $12\frac{1}{2}$ per cent of it, and fed the rest to his stock. How many bushels did he use?

12. Austin picked and sold 80 quarts of berries. $37\frac{1}{2}$ per cent were blueberries, 20 per cent were blackberries, 30 per cent were strawberries, and $12\frac{1}{2}$ per cent were raspberries. How many quarts of blueberries? Blackberries? Strawberries? Raspberries?

13. If a dealer pays 25 cents a yard for cloth, for how much must he sell it to gain 20 per cent?

14. By selling cloth at 25 cents a yard a merchant makes 25 per cent. Find the cost.

WRITTEN PROBLEMS

1. A farmer sold 35 pecks of peas in one week and 40 % as many in the following week. How many pecks in the second week? In both weeks?

2. Out of a case of 4 dozen jars of fruit $\frac{1}{2}$ of a dozen jars were broken. What per cent of the lot was salable?

3. A bicycle that cost $15.75 was sold for 68 % of its cost. How much was received for it?

4. Twenty-five per cent of a man's salary of $1200 is spent for food. How much is spent for food?

5. The carpenter made me a table. He charged $5.75 for materials and $6.25 for labor. I sold the table for $18. What per cent of the cost did I gain?

6. A house that cost $2750 was sold for 18 % less than the cost. How much was received for it?

7. A merchant sold 16 tons of wheat in May and 85 % as much in June. How many tons in June?

8. Two hundred cases of eggs were bought at $9 a case and sold at a gain of 20 %. How much was gained?

9. A piece of silk containing 48 yards and costing $2.50 a yard was sold at a gain of 22 %. What was the gain?

10. Mr. Parsons bought a horse for $150. After keeping it 10 weeks at a cost of $5 a week, he sold it at a gain of 10 % of the entire cost. How much did he gain? What was the selling price?

11. Out of 350 bushels of tomatoes, 280 bushels were *sold.* What per cent was sold?

DIFFERENCE IN TIME BETWEEN DATES

1. Name the months of the year. How many?

2. Name the first month; the fourth; the seventh; the tenth.

3. What name is given to the second month? The fifth? The eighth? The eleventh?

4. What is the third month? The sixth? The ninth? The twelfth?

5. What month of the year is March? November? July? May?

6. How many years, months, and days from Sept. 20, 1906, to April 15, 1910?

First Method:

Years	Months	Days
1910	4	15
1906	9	20
3	6	25

The minuend is the 1910th year, the 4th month, the 15th day. The subtrahend is the 1906th year, the 9th month, the 20th day.

Since we cannot take 20 days from 15 days, we take 1 of the 4 months, express it as 30 days, and put it with the 15 days, making 45 days. 20 days from 45 days leaves 25 days.

Since 9 months cannot be taken from 3 months, we take 1 of the 1910 years, or 12 months, and add it to the 3 months, making 15 months. 9 months from 15 months leaves 6 months.

1906 years from 1909 years leaves 3 years.

From Sept. 20, 1906, to April 15, 1910, is 3 years 6 months 25 days.

In finding the difference in time between two dates in this way, a month is regarded as 30 days.

Second Method:

From Sept. 20, 1906, to Sept. 20, 1909, is 3 years.

From Sept. 20, 1909, to March 20, 1910, is 6 months.

From March 20, 1910, to April 15, 1910, is 26 days.

From Sept. 20, 1906, to April 15, 1910, is 3 years 6 months 26 days.

This method is used when the exact number of days less than a month is required.

TO THE TEACHER: Teach the method in general use among business men in your locality.

Find the difference in time in years, months, and days:

7. From Nov. 17, 1900, to May 30, 1904.

8. From July 20, 1906, to Dec. 25, 1909.

9. From Aug. 6, 1903, to Jan. 1, 1908.

10. From May 24, 1907, to April 3, 1910.

11. From Sept. 12, 1905, to July 7, 1907.

12. The battle of Concord and Lexington occurred April 19, 1775. Cornwallis surrendered at Yorktown Oct. 19, 1781. What was the difference in time between these events?

13. Washington was born Feb. 22, 1732, and became President April 30, 1789. How old was he when inaugurated?

14. Jackson fought the battle of New Orleans, Jan. 15, 1815. He became President March 4, 1827. How long after the battle?

15. The colonies declared their independence July 4, *1776.* How long ago?

16. Daniel Webster was born Jan. 18, 1782. How old was he at the time of his famous speech in the Senate, March 7, 1850?

17. Nellie was born Nov. 17, 1905, and entered the kindergarten Sept. 9, 1910. At what age?

18. How old is a boy who was born May 12, 1898?

19. What is your age in years, months, and days?

Find the difference in time in years, months, and days between these events and today:

20. The landing of the Pilgrims, Dec. 20, 1620.

21. The discovery of America by Columbus, Oct. 12, 1492.

22. The Treaty of Peace at the close of the Revolutionary war, Sept. 3, 1783.

23. The Battle of Bunker Hill, June 17, 1775.

24. The first message by telegraph, May 24, 1844.

25. The birth of Lincoln, Feb. 12, 1809.

INTEREST *Oral*

Mr. Mason has an automobile worth $3000. He lets me have the use of it one hour for $5.

Mr. Harrison has a house worth $3000. He lets me have the use of it one year for $300.

If Mr. Mason or Mr. Harrison should let me have the use of $3000 for a year, would he expect me to pay for the use of it?

Whenever one person has the use of another person's property — his automobile, his house, his money — he pays him for it.

Mr. Ellis let me have $100 for a year. At the end of the year I paid him 6% of $100, or $6, for the use of it.

Money paid for the use of money is *interest.*

The money on which interest is paid is the *principal.*

The per cent of the principal paid each year for interest is the *rate of interest.*

1. What is the interest of $100 for 1 year at 3%? At 4%? At 5%? At 7%? At 8%? At 10%?

2. At 5%, how much must be paid for the use of $200 for 1 year? 2 years? 4 years? 5 years?

Find the interest for 1 year on: *Written*

3. $50 at 4%	4. $60 at 6%	5. $80 at 3%
6. $200 at 5%	7. $400 at 8%	8. $600 at 7%
9. $300 at 7%	10. $500 at 4%	11. $700 at 5%
12. $800 at 6%	13. $900 at 5%	14. $900 at 8%

At 4%, what is the interest on:

15. $200 for 1 yr.?	$1\frac{1}{2}$ yr.?	$1\frac{1}{4}$ yr.?	$1\frac{3}{4}$ yr.?
16. $300 for 1 yr.?	$1\frac{1}{3}$ yr.?	$1\frac{2}{3}$ yr.?	$2\frac{1}{3}$ yr.?
17. $400 for $1\frac{1}{6}$ yr.?	$1\frac{5}{6}$ yr.?	$2\frac{1}{3}$ yr.?	$2\frac{3}{4}$ yr.?

18. What part of a year is 6 months? 3 months? 9 months? 4 months? 8 months? 2 months? 10 months? 1 month? 5 months? 7 months? 11 months?

19. At 4% a year, what part of a year's interest ought to be paid for the use of $300 for 3 months? What interest should be paid?

At 6%, what is the interest on:

20. $800 for 3 months?	21. $700 for 2 months?
22. $500 for 4 months?	23. $400 for 10 months?
24. $300 for 8 months?	25. $400 for 7 months?

Find the interest on:

26. $50 at 4% for 1 year 6 months.

27. $600 at 4% for 1 year 7 months.

28. $80 at 5% for 2 years 3 months.

29. $600 at 6% for 2 years 1 month.

30. What is the interest on $132 for 2 years 7 months at 4%?

$132
.04
$5.28 int. for 1 year
$2\frac{7}{12}$
$3.08 int. for $\frac{7}{12}$ of a year
10.56 int. for 2 years
$13.64 int. for 2 years 7 months

The interest for 1 year is 4% of $132, or $5.28.

The interest for 2 years 7 months is $2\frac{7}{12}$ times $5.28, or $13.64.

Find the interest on:

31. $180 for 3 years at 5%; for 2 years at 4%.

32. $225 for 2 years 4 months at 4%.

33. $420 for 3 years 3 months at 6%.

34. $360 for 2 years 10 months at 7%.

	Principal	Time	Rate	Interest
35.	$640	2 years 5 months	3%	?
36.	$256	3 years 8 months	6%	?
37.	$576	4 years 3 months	5%	?
38.	$900	2 years 4 months	7%	?
39.	$372	1 year 1 month	4%	?
40.	$324	5 years 6 months	5%	?
41.	$456	3 years 8 months	4%	?
42.	$864	4 years 11 months	7%	?
43.	$375	2 years 9 months	6%	?
44.	$264	1 year 9 months	8%	?

. What is the interest on $240 at 5% from Jan. 1 to July 1?

46. How much must be paid for the use of $600 at 7% from Sept. 1 to Jan. 1?

47. On June 1 Mr. Nichols borrowed $225 at 6%. What interest was due Oct. 1? If Mr. Nichols paid both principal and interest Oct. 1, what *amount* did he pay?

The sum of the principal and the interest is the *amount.*

48. On Jan. 1 Mr. Stevens bought a horse for $150, agreeing to pay for it in 6 months with interest at 5%. When must the money be paid? How much?

49. When money is worth 7%, what must be paid to settle in full a loan of $456 for 2 months?

50. Mr. Young bought a house for $3000. He paid down $\frac{2}{5}$ of the price and promised to pay the rest in 4 months with interest at 5%. How much did he pay at time of settlement?

REVIEW EXERCISE

	A		*B*		*C*		*D*	
1.	$\frac{7}{8}$	$\frac{5}{16}$	$2\frac{1}{2}$	$\frac{2}{5}$	$4\frac{1}{8}$	$1\frac{3}{8}$	$4\frac{3}{4}$	$2\frac{8}{15}$
2.	$\frac{15}{16}$	$\frac{3}{4}$	2	$\frac{3}{4}$	$4\frac{1}{4}$	$2\frac{1}{2}$	$3\frac{3}{5}$	$3\frac{1}{8}$
3.	$\frac{3}{4}$	$\frac{1}{6}$	$2\frac{1}{4}$	$\frac{2}{3}$	$3\frac{1}{5}$	$2\frac{1}{2}$	$8\frac{2}{5}$	$6\frac{3}{7}$
4.	$\frac{5}{7}$	$\frac{1}{4}$	$4\frac{2}{5}$	$\frac{3}{5}$	$7\frac{3}{7}$	$1\frac{3}{4}$	$5\frac{2}{5}$	$1\frac{4}{5}$
5.	$\frac{5}{12}$	$\frac{2}{9}$	$2\frac{2}{3}$	$\frac{8}{9}$	$6\frac{1}{4}$	$1\frac{7}{8}$	$6\frac{5}{6}$	$2\frac{1}{9}$
6.	$\frac{5}{8}$	$\frac{4}{7}$	$3\frac{1}{2}$	$\frac{6}{7}$	$5\frac{1}{3}$	$1\frac{1}{9}$	$8\frac{3}{4}$	$3\frac{2}{3}$
7.	$\frac{21}{40}$	$\frac{3}{8}$	$3\frac{1}{3}$	$\frac{9}{10}$	$8\frac{2}{5}$	$6\frac{1}{7}$	$7\frac{2}{9}$	$2\frac{1}{4}$
8.	$\frac{3}{4}$	$\frac{8}{15}$	$5\frac{4}{9}$	$\frac{5}{8}$	$2\frac{5}{8}$	$1\frac{3}{4}$	$9\frac{3}{8}$	$8\frac{1}{3}$
9.	$\frac{6}{7}$	$\frac{5}{6}$	$4\frac{3}{8}$	$\frac{5}{7}$	$3\frac{3}{4}$	$2\frac{4}{5}$	$5\frac{7}{12}$	$2\frac{4}{15}$
10.	$\frac{7}{15}$	$\frac{3}{20}$	$8\frac{5}{9}$	$\frac{7}{8}$	$2\frac{3}{8}$	$1\frac{7}{12}$	$7\frac{2}{7}$	$2\frac{1}{8}$

1. Add the numbers in each couplet.

2. Subtract the second term of each couplet from the first term.

3. Find the product of the terms in each couplet.

4. Divide the first term of each couplet by the second term.

5. Divide the second term of each couplet by the first term.

6. Make up problems based on the combinations in the table.

MISCELLANEOUS PROBLEMS

1. The rainfall in a city in one year was as follows:

January,	3.386 in.
February,	3.957 in.
March,	5.900 in.
April,	5.691 in.
May,	0.818 in.
June,	9.180 in.
July,	2.621 in.
August,	3.676 in.
September,	1.745 in.
October,	4.461 in.
November,	1.527 in.
December,	3.011 in.

(1) What was the total rainfall for the year?

(2) What was the average rainfall per month?

(3) How much more rain fell in June than in July?

(4) How much more rain fell in the first half of the year than in the last half of the year?

(5) What amount fell in the three months of least rainfall? In the three months of greatest rainfall?

2. Four and one half acres of land were cut up into 24 house lots. What part of an acre was each lot?

3. Mr. Mason paid $240 for a carriage and sold it for 80 % of what he paid. For how much did he sell it?

4. What per cent of the cost is loss when hay is bought at $20 a ton and sold at $18 a ton?

5. The area of Alaska is about 600,000 square miles. We paid Russia $7,200,000 for it. How much per square mile? Per acre?

6. A man spends $\frac{7}{8}$ of his salary, and saves $280. What is his salary?

7. A earns $2\frac{1}{4}$ per day, and spends $1\frac{3}{4}$. How many days must he work to save $100?

8. A man owing a debt of $280 paid 40% of it at one time, and 50% of the remainder at another time. How much did he still owe?

9. How many articles, at the following rates, can be purchased with a $5 bill: $12\frac{1}{2}$¢? $16\frac{2}{3}$¢? 50¢? $33\frac{1}{3}$¢? $8\frac{1}{3}$¢? $6\frac{1}{4}$¢? 25¢? $3\frac{1}{3}$¢? $6\frac{2}{3}$¢?

10. How many yards of ribbon at $12\frac{1}{2}$¢ a yard are equal in value to 24 yards worth $33\frac{1}{3}$¢ a yard?

11. A fruit dealer bought a box of oranges for $2.50. He sold them for $3.50. What part of the cost did he gain? What per cent?

12. Add the quotients of $.1152 \div 12$ and $11.52 \div 12$.

13. Find the difference between the quotient of $.012 \div 4$ and the product of $.012 \times 4$.

14. Multiply the quotient of $.08 \div 4$ by the sum of 2.12 and .88.

15. Divide the sum of 2.2 and .2 by their difference.

16. Mr. Hodge's bill for 15 days' work in making a farmer's wagon was $35.25. How much did he charge *per day*?

17. The freight charge between two places is 58 cents per hundred pounds. What is the charge for shipping 20 barrels of oil, each weighing 310 pounds?

18. A Kansas farmer bought 80 acres of cheap land for $240. Oil being found on his farm, he sold his land for $60,000. What was his profit? How much did the land cost per acre? For how much was it sold per acre?

19. A grocer bought 210 pounds of tea for $100.80, and sold the lot for $117.60. What was the gain per pound?

20. At another time he bought 87 pounds for $46.11. He sold this at a gain of 6 cents a pound. How much did he receive for the lot?

21. One hundred fifty pounds of butter cost $49.50. It was sold at 38 cents a pound. What was the gain?

22. A field 120 rods by 40 rods produced 28¾ bushels of wheat to the acre. What was the crop worth at 92 cents a bushel?

23. A dealer mixed 6 pounds of coffee costing 24 cents a pound and 12 pounds costing 30 cents a pound. What was the value of one pound of the mixture?

24. Mr. Burns sold to Mr. Dean 36 acres of land at $56 an acre. Mr. Dean gave in payment $639 in money and a city house lot. What was the value of the house lot?

25. Mr. Cushing raised 440 bushels of potatoes. Sixty-five bushels were unmarketable. He sold the rest at 75 cents a bushel. How much did he receive?

26. What is the gain on 36 bunches of bananas bought at $1.37 each and sold at $1.95 each?

27. A merchant bought eggs at 32 cents a dozen and sold them at 40 cents a dozen. His gain was $2.16. How many dozen did he buy and sell?

28. A merchant made $9.88 by buying roasting pans at 37 cents each and selling at 50 cents each. How many did he buy and sell?

29. Bronze is composed of 18 parts copper, 1 part tin, 1 part zinc.

(1) What per cent of each metal is used?

(2) How many pounds of each metal in a bronze statue weighing 2000 pounds?

30. In 3.2 hours a man walks 12.32 miles. How far does he walk in one hour?

31. If a man can walk 3.2 miles in one hour, how many miles can he walk in 12.32 hours?

32. If you ride your wheel for an hour at the rate of 12.32 miles per hour, how many miles have you to ride after going 3.2 miles?

33. John rode his wheel 12.32 miles on Tuesday and 3.2 miles on Wednesday. How far did he ride on both days?

34. On Thursday he rode 12.32 miles, and on Friday 3.2 miles more than on Thursday. How far did he ride on both days?

35. What per cent of the cost is the gain when flour is bought for $6 a barrel and sold for $7 a barrel?

36. What is the gain per cent in buying flour at $6.00 a barrel and selling at $7.50 a barrel?

37. If a man sells $\frac{3}{7}$ of his farm of 175 acres, how many acres has he left?

38. What is the greatest number that will divide all of these numbers, — 108, 144, and 168?

39. A workman receives $15.75 per week. He spends $3.25 for rent, $3.15 for groceries, $3.70 for provisions, $2.37 for clothing, $0.63 for luxuries, and deposits $2 in the bank. How much has he left?

40. At $75 per acre, what is the value of a lot of land 65 rods by 48 rods?

41. Four boys go camping for a week. They pay 50¢ a day for their tent, $1.60 a day for their living, 50¢ a day for their boat, and each pays 75¢ for car fare. How much did each pay for his outing?

42. Mr. Arthur R. Bean rents a house to Mr. Charles L. Roper for $18 a month. Make out a receipt for rent received for the month of July, 1912.

43. On April 10, 1911, Mrs. A. M. Baker buys the following goods from Gardner and Bassett: $3\frac{1}{2}$ yd. lawn at 20¢ a yard, 3 yd. table damask at $1.25 a yard, and $1\frac{1}{2}$ doz. towels at $12\frac{1}{2}$¢ each. Make out her bill.

44. The stripes of the flag on the schoolhouse are $2\frac{3}{4}$ inches wide. What is the width of the flag?

45. A piece of cardboard 12 in. square is cut into pieces 3 in. by $1\frac{1}{2}$ in. How many?

46. A house lot is 68 ft. long and $\frac{3}{4}$ as wide. What is its perimeter?

47. A club bought 20 baseballs, paying at the rate of $15 a dozen. How much?

48. What are the prime factors of 168?

49. Change $\frac{84}{210}$ to lowest terms.

CROPS RAISED BY A STATE SCHOOL *Written*

A state school raised the following crops on its farm. Find the value of each crop at the given rate. (Keep answers.)

	Crop	Quantity	Market Rate
1.	Apples	1,124 barrels	$1.95 per bbl.
2.	Beets	1,373 bushels	50¢ per bu.
3.	Cabbage	54,396 pounds	1¢ per lb.
4.	Carrots	1,660 bushels	45¢ per bu.
5.	Corn (ensilage)	545 tons	$12.50 per T.
6.	Corn (fodder)	147 tons	$10.25 per T.
7.	Corn (green)	302 bushels	55¢ per bu.
8.	Cucumbers	110 boxes	$5 per box
9.	Lettuce	194 boxes	40¢ per box
10.	Onions	408 bushels	75¢ per bu.
11.	Potatoes	9,659 bushels	67¢ per bu.
12.	Pumpkin	17,028 pounds	$\frac{1}{2}$¢ per lb.
13.	Rhubarb	2,948 pounds	$2\frac{1}{2}$¢ per lb.
14.	Squash (summer)	94 barrels	70¢ per bbl.
15.	Squash (winter)	20 tons	1¢ per lb.
16.	Tomatoes (green)	120 bushels	40¢ per bu.
17.	Tomatoes (ripe)	92 bushels	60¢ per bu.
18.	Turnips	768 barrels	$1.15 per bbl.

Find the total market value of the crops.